Essays on Incongruent Preferences for Effort Allocations in Multi-Task Agency Relations

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Abstract

This thesis comprises three self-contained essays that deal with inefficient effort allocations in multi-task agency relations with moral hazard.

The first essay analyzes a multi-task agency framework where the agent exhibits task-specific abilities. Besides investigating the appendant consequences of applying incongruent performance measures in incentive contracts, this essay demonstrates that the provision of incentives—including the optimal aggregation of information—takes the agent's task-specific abilities into consideration. It further emphasizes the relation between job characteristics and the principal's preference for selecting specific agents. This essay essentially demonstrates that differences in task-specific abilities across agents can explain why they are allocated to various jobs; or why they receive different incentive contracts, even if their jobs are identical.

The second essay considers a multi-task agency model with a risk-neutral and financially constrained agent. It analyzes the inefficiency of the agent's induced effort allocation across tasks when the principal has only access to an incongruent information system about the agent's performance. This essay further investigates the costly acquisition of information aimed at improving the agent's performance evaluation, and therefore, mitigating her effort distortion. It contrasts two alternatives for the principal: (*i*) to centrally invest in the information acquisition; or (*ii*), to delegate this task to a supervisor. This essay demonstrates that the principal generally favors delegation for a sufficiently incongruent information system, and a centralized investment, otherwise. This can be observed if the supervisor's performance evaluation is adequately precise. Otherwise, the contrary implication applies.

The third essay analyzes incongruent preferences between firms for the characteristics of exchanged goods as an inefficiency of mutual market transactions, and collusive behavior within firms as an inefficiency of integrated transactions. It investigates the consequences of both inefficiencies on (*i*) firms' decision on whether to integrate transactions or to utilize the market; and (*ii*), the properties of contractual arrangements. This essay proposes two important implications. First, diverging preferences between firms affect diametrically their benefits of mutual market transactions. Better aligned preferences are thereby disadvantageous from the demanding firm's perspective since it imposes higher costs to ensure relation-specific investments. Second, anticipated collusion may force firms to employ market transactions, even if integrated transactions would have been more efficient otherwise. Nevertheless, potential collusion can be beneficial if it facilitates the achievement, or improves the efficiency, of superior relational contracts within and between firms due to a worse fall-back alternative.

Keywords:
Multi-task Agencies, Performance Measurement, Effort Distortion, Congruity, Incentives

Zusammenfassung

Diese Dissertation enthält drei eigenständige Aufsätze, welche sich mit Prinzipal-Agenten Beziehungen in Verbindung mit moralischem Risiko beschäftigen. In diesem Zusammenhang ist der Agent für die Ausführung von multiplen Aufgaben (Multitasking) verantwortlich.

In dem ersten Aufsatz wird ein Prinzipal-Agenten Modell mit multidimensionaler Arbeitsanstrengung des Agenten analysiert, wobei angenommen wird, dass der Agent unterschiedliche Fähigkeiten für die Ausführung der einzelnen Aufgaben aufweist. Dabei werden die durch die Anwendung von inkongruenten Leistungsmaßen in Anreizverträgen resultierenden Ineffizienzen in Abhängigkeit von den aufgabenspezifischen Fähigkeiten des Agenten identifiziert und analysiert. Der Aufsatz zeigt dabei, dass ein optimales Anreizsystem—einschließlich der effizienten Aggregation von verfügbaren Informationen—auf die aufgabenspezifischen Fähigkeiten des Agenten abgestimmt ist. Der Aufsatz stellt weiterhin dar, wie die Eigenschaften eines Arbeitsplatzes die optimale Auswahl von heterogenen Agenten bestimmt. Es wird dabei aufgezeigt, dass Divergenzen bei aufgabenspezifischen Fähigkeiten erklären können, warum heterogene Agenten auf unterschiedliche Arbeitsplätze verteilt werden. Darüber hinaus können aufgabenspezifische Fähigkeiten begründen, weshalb Anreizverträge zwischen Agenten variieren, selbst wenn ihre Aufgabenbereiche identisch sind.

Der zweite Aufsatz betrachtet ein Prinzipal-Agenten Modell mit einem risikoneutralen und haftungsbeschränkten Agenten, wobei der Agent wiederum für die Ausführung von multiplen Aufgaben verantwortlich ist. In diesem Modellrahmen wird die Ineffizienz der induzierten Verteilung der mehrdimensionalen Arbeitsanstrengung auf die relevanten Aufgaben analysiert, die durch die Verfügbarkeit eines nur inkongruenten Informationssystems über die Leistung des Agenten resultiert. Dieser Aufsatz analysiert weiterhin die Generierung von zusätzlichen Leistungsmaßen mit dem Ziel, die Verzerrung der multidimensionalen Arbeitsanstrengung des Agenten zu reduzieren. In diesem Aufsatz werden dabei zwei Alternativen für den Prinzipal herausgearbeitet und verglichen: (*i*) zentral in die Leistungsmessung des Agenten zu investieren, oder (*ii*), diese Aufgabe an einem Vorgesetzten (Supervisor) des Agenten zu delegieren. Es wird dabei gezeigt, dass der Prinzipal die Delegation dieser Aufgabe bevorzugt, sofern das bereits verfügbare Informationssystem in hohem Maße inkongruent ist. Ist das Informationssystem dagegen hinreichend kongruent, ist im Gegenzug eine zentrale Investition in die zusätzliche Leistungsmessung effizienter. Diese Beobachtung ergibt sich unter der Bedingung, dass ausreichend präzise Informationen über die Arbeitsanstrengung des Vorsetzten verfügbar sind. Andernfalls können die entgegengesetzten Resultate beobachtet werden.

Der dritte Aufsatz analysiert inkongruente Präferenzen zwischen verschiedenen Unternehmen bezüglich der Eigenschaften der zwischen ihnen ausgetauschten Güter als mögliche Ineffizienz von Markttransaktionen. Der Aufsatz berücksichtigt darüber hinaus die Möglichkeit von kollusivem Verhalten innerhalb von Unternehmen als mögliche Ineffizienz von integrierten Produktionen. Dabei erfolgt eine Untersuchung der Auswirkungen dieser Ineffizienzen hinsichtlich (*i*) der Entscheidung von Unternehmen, erforderliche Transaktionen intern zu organisieren, oder hierfür den Markt in Anspruch zu nehmen, und (*ii*), der sich daraus ergebenden Eigenschaften der vertraglichen Vereinbarungen innerhalb und zwischen Unternehmen. Die Analysen in diesem Aufsatz führen dabei zu zwei wichtigen Schlussfolgerungen. Erstens, zwischen Unternehmen divergierende Präferenzen hinsichtlich der Eigenschaften von ausgetauschten Gütern haben einen umgekehrten Effekt auf die jeweilige Rentabilität von bilateralen Transaktionen. Dabei sind stärker übereinstimmende Präferenzen nachteilig aus der Sicht des nachfragenden Unternehmens, weil dies höhere implizite Kosten für die Sicherstellung von beziehungsspezifischen Investitionen verursacht. Zweitens, antizipierte interne Kollusion kann dazu führen, dass Unternehmen erforderliche Güter über den Markt beschaffen, obwohl eine interne Produktion unter Abwesenheit von kollusivem Verhalten effizienter sein würde. Im Gegensatz dazu kann antizipierte Kollusion auch vorteilhaft sein, sofern es durch eine schlechtere Alternative die Einhaltung von profitableren relationalen Verträgen innerhalb und zwischen Unternehmen ermöglicht bzw. deren Effizienz erhöht.

Schlagwörter:
Mehraufgaben Prinzipal-Agenten Beziehung, Leistungsmessung,
Anstrengungsverzerrung, Kongruenz, Anreize

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Introduction

Employees are generally charged with performing a collection of various tasks that contribute to firm value differently. The accountability for multiple tasks implies that employees can not only decide on their effort intensity, but also on how to allocate their effort across these tasks. This in turn provides them with significant latitude to place more emphasis on certain tasks relative to others. From firms' perspective, it would be desirable to assign effort to particular tasks in accordance to their relative contribution to firm value so as to ensure the implementation of an efficient effort allocation across all relevant tasks. However, many economic relationships are subject to moral hazard, i.e. effort is non-contractible such that firms face an incentive problem. Firms are therefore obliged to apply appropriate incentive mechanisms to motivate their employees to implement effort.

One frequently applied mechanism is the provision of incentive contracts on the basis of objective performance measures. As shown by previous agency literature, the application of verifiable but imprecise measures about an agent's effort leads to suboptimal (second-best) contracts, whenever the agent is either risk-averse or financially constrained.[1] Holmström and Milgrom [1991] exposed an additional inefficiency when agents can determine their effort allocation across tasks: the provision of incentives based on performance measures leads to individual preferences for particular tasks, thereby potentially inducing an inefficient effort allocation from the principal's perspective. Such *effort distortion* occurs if the individual performance evaluation does not perfectly reflect the agent's contribution to firm value [Feltham and Xie, 1994]. Generally speaking, the agent might be motivated to place more emphasis on less valuable tasks relative to tasks with higher contributions to firm value. In some cases, agents focus on activities which have little or even negative effects on firm value, but are suitable to improve their performance evaluation.

For example, by the spring of 2002, almost every state in the U.S. implemented school-level tests, initially aimed at comparing students' performances across public schools.[2] To improve teaching quality, some states further utilized these school-level tests for rewarding or sanctioning schools and/or individual teachers. The basis for such evaluations is grounded in the belief that students' performance in these tests appropriately reflects teaching quality, i.e. the better teachers are in conveying the required knowledge and improving their students' skills, the higher would be their

[1] For a review of agency literature dealing with moral hazard, refer e.g. to Eisenhardt [1989] for an early survey, and for more recent surveys, to Prendergast [1999], Lambert [2001], Gibbons [2005], and Christensen and Feltham [2005].

[2] See Kane and Staiger [2002] for a detailed description of school-level tests in the U.S. and their differences among states.

students' test score. For instance, the state California spent almost $700 million for financial incentives in 2001, whereby teachers could get up to $25,000 of bonuses in schools with the highest performance improvement [Kane and Staiger, 2002]. However, Kane and Staiger [2002] raised concerns about the reliability of school-level tests since they represent only a small fraction of teaching responsibilities such as teaching basic rather than advanced knowledge; or they tend to focus on particular subjects like Math or English, but disregard other essential sciences. Moreover, Linn [2000] cautioned against potential undesired consequences if school-level tests are used as incentive devices.

Since empirical studies indicate that individuals are highly responsive to monetary incentives (e.g. Asch [1990], Paarsch and Shearer [1999] and Lazear [2000a]), it is predictable that the provision of incentives based on students' achievements in school-level tests motivates teachers to shift their teaching emphasis to the knowledge and skills covered by the respective tests. In fact, Stecher and Barron [1999] provide evidence of teachers in Kentucky spending more time on science in fourth grade when science was tested; and on Math in fifth grade when Math was tested. Other studies provide further evidence that teachers shifted their teaching emphasis on tested, at the expense of non-tested knowledge, see e.g. Klein et al. [2000] and Jacob [2002].

However, some teachers did not only shift their effort allocation to measured and therefore, rewarded tasks, but also began to manipulate their students' test scores in order to enhance their own performance evaluation. For instance, Goodnough [1999] reported that in New York City alone, several teachers were accused of manipulating the test scores by encouraging their students to change incorrect answers, or by incorporating questions from the actual test in their practice tests.[3] Jacob and Levitt [2003] estimate that at least four percent of classroom tests in Chicago elementary schools were affected by such manipulations. They further indicated that the frequency of manipulations is highly responsive to minor changes in offered incentive schemes.

As the above example demonstrates, implementing a reward scheme—initially intended to motivate teachers—resulted in two inefficiencies. First, teachers shifted their teaching emphasis on tested, at the expense of untested knowledge and skills. Generally speaking, they *distorted* their effort allocation across all relevant tasks with the objective of improving their own performance evaluation. Second, the provision of incentives based on students' achievements in school-level tests induced, in some cases, dysfunctional behavior: some teachers have chosen actions (manipulation) which are suitable to enhance their performance evaluation, but do not contribute to the objective of educating students. Of course, the latter inefficiency applies only to a small fraction of teachers. However, the first inefficiency—shifting the teaching emphasis on measured knowledge and skills—also constitutes a serious drawback of utilizing test scores as incentive devices since students are eventually less educated in non-tested, but vital fields. Yet ironically, teachers did what they are rewarded for: improving their students' test-scores. To achieve this objective, they adjusted their effort allocation appropriately—a response to the implemented incentive scheme, which was not intended.

[3] For further examples of encountered manipulations see Tysome [1994] and Hofkins [1995].

The preceding example illustrates how the provision of incentives on the basis of performance measures may induce effort distortion if they do not accurately reflect the agent's contribution to the objective of organizations.[4] The first analytical investigation of this phenomenon by Holmström and Milgrom [1991] attracted the attention of many researchers who also delved into multi-task agency relationships by investigating incentive contracts, which aim at ensuring optimal effort allocations in addition to balancing incentives and the agent's desire for insurance. Other important work include Feltham and Xie [1994] and Datar et al. [2001] who, in particular, focus on the aggregation of performance measures with the objective of improving incentive contracts in multi-task agencies. However, there are several issues which have been not addressed so far, but are essential for our appreciation of the nature of contracts in multi-task agency relations. This thesis thus contributes to contemporary multi-task literature by addressing important issues in agency relations, when the agent can not only decide on her effort intensity, but also on her relative effort allocation.

Particularly, this thesis comprises three self-contained essays dealing with multi-task agency relations. The first essay analyzes the provision of incentives when agents exhibit task-specific abilities. It illustrates how incentive contracts are adjusted to these agent-specific characteristics. The second essay analyzes costly performance measurement with the objective of providing the agent with more congruent incentives, and therefore, mitigating effort distortion. The main emphasis is on the comparison between a centralized versus a delegated generation of additional performance measures. The third essay considers incongruent preferences for effort allocations between firms as an inefficiency to employ the market. It contrasts this inefficiency to potential collusion within firms as an inefficiency of integrated transactions. Subsequently, I dwell on each essay and summarize their main results.

In the first essay, I analyze a multi-task agency framework with a risk-neutral principal and a risk-averse agent. The principal has only access to performance measures which do not perfectly reflect the agent's individual contribution to firm value, i.e. these measures are incongruent. Moreover, the agent is characterized by different task-specific abilities, i.e. she can perform some tasks more efficiently than others. Besides the characteristics of applied performance measures, diverging ease for performing relevant tasks additionally affect the agent's preference for allocating her effort across tasks. The main emphasis of this essay is on whether and how incentive contracts are adjusted to agents' task-specific abilities in combination with the characteristics of relevant tasks and available performance measures. This essay further elaborates on criteria which are sufficient to compare information systems in multi-task agencies.

The first essay demonstrates that the provision of incentives incorporates the agent's task-specific abilities. This implies that different agents generally receive diverging incentive contracts, even if they are in charge of performing identical tasks. It further illustrates that the signal/noise ratio—a sufficient criteria to rank performance measures in single-task agencies—is only applicable in multi-task agencies, when performance measures provide identical information about the agent's relative

[4]For further illustrative examples refer to Kerr [1975], Gibbons [1998], Prendergast [1999], and Baker [2000].

effort allocation across tasks. If performance measures offer different information about the implemented effort allocation, their relative value additionally depends on their respective congruity in conjunction with the agents' tasks-specific abilities.

This essay further analyzes how multiple performance measures are aggregated to improve the contract efficiency. Besides reducing the incentive risk imposed on the risk-averse agent, the additional purpose of the optimal aggregation is to motivate a more efficient effort allocation. As demonstrated, the efficient aggregation depends on the agent's particular abilities, thereby implying that different agents are evaluated by diverse performance measurements, even if they are in charge of performing identical tasks. This essay further illustrates that the principal can induce the first-best effort allocation if she receives a sufficient quantity of appropriate performance measures. Nonetheless, this is only efficient if aggregating performance measures with the objective of motivating the first-best effort allocation contemporaneously minimizes the imposed incentive risk. Finally, this essay elaborates on adverse selection and highlights the relation between job characteristics and the principal's preference for selecting specific agents. It indicates that discrepancies in task-specific abilities across agents can explain why they are allocated to various jobs; or why they receive different incentive contracts, even if their jobs are identical.

The second essay considers a multi-task agency relationship with a risk-neutral and financially constrained agent. The principal has only access to an incongruent information system, i.e. it does not reflect the agent's contribution to firm value. The application of this information system to provide the agent with incentives motivates her to implement an inefficient effort allocation across relevant tasks. The purpose of this essay is the investigation of costly performance measurement aimed at improving the incentive congruity, and therefore, mitigating effort distortion. It contrasts two alternatives for the principal: (*i*) to centrally invest in the acquisition of additional measures about the agent's performance; and (*ii*), to recruit a supervisor who is charged with the generation of these measures. However, employing a supervisor imposes a second moral hazard problem.

This essay first analyzes the consequences of providing a financially constrained agent with a bonus contract on the basis of incongruent performance evaluations. It illustrates that the agent extracts an economic rent, which is increasing in the degree of congruence of her performance evaluation. This further suggests that improving the congruence of her performance evaluation with the objective of mitigating her effort distortion incurs implicit costs due to a higher rent.

The second essay demonstrates that the principal's decision on whether to centralize or to delegate the information acquisition depends on the relationship between three factors: (*i*) the precision of the supervisor's performance evaluation, (*ii*) the supervisor's comparative cost advantage in generating the required information; and (*iii*), the congruence of the costless available information system about the agent's effort. If the supervisor's performance evaluation is adequately precise, this essay provides two critical implications. First, a sufficiently incongruent costless information system generally favors delegation. This is because a less congruent costless information system imposes lower requirements on the supervisor's relative measurement efficiency, which is more likely to be provided by a potential supervisor. Second, the more congruent the costless information system is, the more likely is a centralized investment in the information acquisition. The rationale is that delegation—in order to be preferred by the principal—imposes high requirements on the supervisor's relative measurement efficiency. This in turn is less likely to be achieved by a potential supervisor. The reverse observations apply if the supervisor's performance evaluation is sufficiently imprecise.

The third essay uses a multi-task agency framework to investigate the properties of transactions within and between firms. Particularly, it considers two alternatives for a firm to obtain a required good: (*i*) through an integrated production; and (*ii*), via a market transaction with another firm. In both cases, the firm can either utilize spot contracts based on contractible information, or non-enforceable relational contracts on the basis of observable but non-contractible information. This essay analyzes how incongruent preferences between firms with respect to the properties of exchanged goods determine the contractual arrangements of their relationship. Tailoring the exchanged good to the demanding firm's needs thereby requires the implementation of a different effort allocation than for selling the good on the market. This potential inefficiency of engaging in market transactions is contrasted to potential side-contracting (collusion) as inefficiency of integrated productions.

This essay demonstrates that incongruent preferences between firms affect the efficiency of market transactions based on relational contracts, which are aimed at ensuring relation-specific investments. The analysis indicates that more congruent preferences between firms are disadvantageous from the demanding firm's perspective. The rationale for this observation is that more congruent preferences impose higher costs for motivating the supplier to make relation-specific investments in the sense of tailoring the exchanged good to the demanding firm's requirements. In contrast, diverging preferences do not affect the value of spot market transactions from the perspective of the demanding firm. This is because the exchange of less suitable goods implies a lower transfer price, which compensates the demanding firm for the lack of perfectly tailored goods.

This essay further illustrates that the effect of potential collusion is ambiguous. First, it can oblige firms to engage in market transactions even though an integrated production would have been more efficient otherwise. Second, anticipated collusion can be advantageous if it deteriorates the efficiency of integrated productions as best fall-back alternative for relational contracts. In this case, it either facilitates the achievement, or improves the efficiency, of superior relational contracts within and between firms. In addition, the third essay considers the consequences on the efficiency of transactions within and between firms, when the demanding firm receives verifiable but incongruent performance measures suitable for improving the efficiency of integrated productions. It illustrates that the availability of sufficiently congruent performance measures generally leads to integrated productions instead of market transactions. Nevertheless, they can also improve the efficiency of integrated productions as best fall-back alternative for relational contracts. As a consequence, access to adequately congruent performance measures can either compromise the feasibility of superior relational contracts within and between firms, or deteriorate their profitability.

Essay 1

Task-Specific Abilities in Multi-Task Agency Relations

1.1 Introduction

Empirical investigations have offered an abundance of evidence suggesting that individuals are highly responsive to monetary incentives (see e.g. Asch [1990], Paarsch and Shearer [1999] and Lazear [2000a]). Nevertheless, the specific effects of reward schemes are somewhat ambiguous when individuals are required to perform a collection of different tasks. In such situations, Kerr [1975] cautioned against the consequences of a reward system that inefficiently overemphasizes some tasks while underemphasizing others. An illustrative example cited by Kerr [1975] is the difficult trade-off between research and teaching responsibilities encountered by faculties at universities. Since teaching quality is harder to assess relative to research output, and prospective promotion decisions mainly hinge on research performance, it is a common phenomenon for faculty members to focus on research at the expense of teaching.[1] Inefficient effort allocations generally occur when the principal is unable to inexpensively access a performance evaluation which perfectly coincides with her objective. If monitoring is too costly, the principal is, to some extent, compelled to accept that an agent is motivated to allocate her effort inefficiently across multiple tasks.

This phenomenon has prompted Holmström and Milgrom [1991] to delve into multi-task agency relationships by investigating incentive contracts which aim at ensuring appropriate effort allocations in addition to countervailing incentive risk and the agent's desire for insurance. Feltham and Xie [1994] also investigate inefficient effort allocations motivated by the application of incongruent performance measures in incentive contracts. According to Feltham and Xie [1994], incongruity arises whenever performance measures do not perfectly reflect the agent's contribution to firm value. They alluded that the agent is only motivated to improve her performance evaluation, thereby leading her to focus on less or even non-valuable tasks, and disregarding more beneficial ones [Feltham and Xie, 1994].[2]

Previous multi-task literature such as Feltham and Xie [1994], Banker and The-

[1] See Brickley and Zimmerman [2001] for an empirical study of this example.

[2] See as well the discussion in Gibbons [1998].

varanjan [2000], and Datar et al. [2001] focus on performance measure congruity and its effects on the efficiency of incentive contracts, but absent from these studies is the possibility that agents may perform some tasks more efficiently than others. Recent literature, however, emphasizes the role of acquiring human capital for specific tasks (see e.g. Lindbeck and Snower [2000], Gibbons and Waldman [2003] and Gibbons and Waldman [2004]).[3] Since individuals differ substantially in their learning aptitudes, which inevitably lead to discrepancies in skills and abilities [Gibbons and Waldman, 2003], it is reasonable to infer that different individuals might perform different tasks with varying degrees of ease.[4] For example, Sapienza and Gupta [1994] indicate in their study of principal-agent relations within venture capital-backed firms that the frequency of venture capitalist (principal) - CEO (agent) interaction is partially dependent on the CEOs' venture experience. They provide evidence that CEOs with prior experiences (i.e. greater proficiency) in start-up ventures would have a lesser tendency of consulting with their venture capitalist.

In order to understand the nature of contracts in multi-task agency relations, it is essential to investigate whether and how task-specific abilities influence the agent's preferences for her effort allocation and the optimal provision of incentives in response to these abilities. This essay thus focuses on multi-task agencies in order to gain new insights into the provision of incentives if performance measures are incongruent with the principal's objective and the agent exhibits different abilities for performing relevant tasks.

This essay investigates how incentive contracts respond to individual task-specific abilities combined with incongruent performance measures. It further demonstrates how the value of performances measures can be compared in multi-task agencies. The analysis indicates that the signal/noise ratio—sufficient to rank performance measures in single-task agencies—can only be applied if all available measures provide the same information about the agent's relative effort allocation. The proposed ranking criteria is in general contingent on the agent's specific abilities such that different agents may imply various orderings of performance measures. This essay further considers the optimal aggregation of multiple performance measures based on the agent's respective task-specific abilities. If the principal has access to a sufficient quantity of appropriate measures, it demonstrates that she can combine them in order to motivate the agent to implement the first-best effort allocation. This, however, is only efficient, if the motivation of the first-best effort allocation by the appropriate aggregation of performance measures contemporaneously maximizes the precision of the information system, which in turn is determined by the agent's task-specific abilities. Finally, this essay illustrates the relevance of adverse selection and highlights the relation between job characteristics and the principal's preference for selecting specific agents.

This essay combines two strands of literature. First, the analyzed framework builds on the multi-task agency model developed by Holmström and Milgrom [1991], and incorporates incongruent performance measures as analyzed by Feltham and Xie

[3]For empirical evidence see Baker et al. [1994b].

[4]Maher et al. [1979] conceive the term 'congruence of perception with preferences' to indicate the phenomenon that even if an individual possesses the correct perception of different tasks, there might still be a preference on specific tasks.

[1994], Baker [2002] and Banker and Thevaranjan [2000]. Second, it incorporates task-specific human capital in the sense of Gibbons and Waldman [1999], Lindbeck and Snower [2000], and Gibbons and Waldman [2003, 2004]. The main contribution of this essay to previous multi-task literature is the incorporation of task-specific abilities and the investigation of their effects on incentive contracts, when the principal receives only incongruent performance measures. It broadens our understanding of incentive contracts in multi-task agency relations by providing three important implications: First, incentive contracts are tailored to the specific abilities of agents, thereby implying that the principal does not generally provide identical incentive contracts when agents differ with respect to their task-specific abilities. Second, the principal's preference for agents with specific abilities depends on the characteristics of relevant tasks and the available information system. Third, the principal can be indifferent between various agents, but may nevertheless provide them with different incentive contracts. In general, different task-specific abilities across agents can explain why they are allocated to various jobs; or why they receive different incentive contracts, even though their jobs are identical.

This essay proceeds as follows. In section 1.2, I give an overview of the model and derive the first-best contract in section 1.3. I provide in section 1.4 the second-best contract and focus on the relation between performance measure congruity and effort distortion in section 1.5. In section 1.6, I investigate how performance measures can be ranked in multi-task agencies, in particular when agents are characterized by task-specific abilities. The optimal aggregation of multiple performance measures as a device to mitigate effort distortion is analyzed in section 1.7. I further investigate the role of adverse selection in section 1.8, and expose the principal's preference for specific agents. Section 1.9 concludes.

1.2 The Model

Consider a single-period agency relationship between a risk-neutral principal and a risk-averse agent. The principal owns an asset and requires the agent's productive effort. Once employed, the agent is in charge of performing $n \geq 2$ tasks (multi-tasking). These tasks are tied together, i.e. the principal cannot split and allocate them to different agents.[5] The agent implements an effort vector $\boldsymbol{e} = (e_1, ..., e_n)^t$, $\boldsymbol{e} \in \mathbf{E} \subseteq \mathbb{R}^{n+}$, where e_i is the agent's effort allocated to task i.[6] Effort is non-verifiable and all activities $e_i \in \mathbf{E}$ are measured in the same unit.

To incorporate task-specific abilities for the agent, I adapt Lazear's [2000b] approach for a single-task agency model to this multi-task framework. In this sense, the abilities differ across tasks and determine the absolute and marginal effort costs borne by the agent. Let $\boldsymbol{\Psi}$ be an $n \times n$ matrix representing the agent's task-specific abilities. The agent's effort costs are contingent on $\boldsymbol{\Psi}$ and take the form $C(\boldsymbol{e}) = \boldsymbol{e}^t\boldsymbol{\Psi}\boldsymbol{e}/2$. For the ease of illustrating the basic relationship between performance measure congruity and effort distortion by using geometric interpretations, I

[5] For considerations on how multiple tasks are efficiently split among several agents, refer e.g. to Holmström and Milgrom [1991], Corts [2005], and Schöttner [2005].

[6] All used vectors are column vectors where 't' denotes the transpose.

assume first that abilities across different tasks are mutually exclusive of one another. Accordingly, $\mathbf{\Psi}$ is a diagonal $n \times n$ matrix defined by $\mathbf{\Psi} = \mathrm{diag}(\psi_1, ..., \psi_n)$, $\psi_i > 0$, $i = 1, .., n$. I will relax this assumption in section 1.7 and allow the agent to feature cost substitutes or complements. A higher ability for performing task i is characterized by a lower ψ_i, $i = 1, ..., n$, and vice versa.[7] I first treat these task-specific abilities as exogenous in order to illustrate the corresponding incentives contracts and induced effort distortions for a given type of agent. However, I will emphasize the principal's preference for employing particular agents by elaborating on adverse selection in section 1.8.

The agent's preferences are represented by the negative exponential utility function

$$U(w, \boldsymbol{e}) = -\exp\left[-\rho\left(w - C(\boldsymbol{e})\right)\right], \tag{1.1}$$

where ρ denotes the Arrow-Pratt measure of absolute risk-aversion and w as the agent's wage. For parsimony, let $\bar{w} = 0$ be her reservation wage implying a reservation utility $\bar{U} = -1$.

By conducting effort $\boldsymbol{e}$, the agent contributes to the principal's non-verifiable gross payoff $V(\boldsymbol{e}) = \boldsymbol{\mu}^t\boldsymbol{e} + \varepsilon_V$, where ε_V is a normally distributed random component with zero mean and variance σ_V^2, representing firm-specific and economy wide risk. The n-dimensional vector $\boldsymbol{\mu} = (\mu_1, ..., \mu_n)^t$, $\mu_i \geq 0$, $i = 1, ..., n$, characterizes the marginal effect of $\boldsymbol{e}$ on gross payoff $V(\boldsymbol{e})$. Since $V(\boldsymbol{e})$ is non-verifiable, it cannot be part of an explicit single-period incentive contract. The only verifiable information about $\boldsymbol{e}$, however, is provided by the performance measure

$$P(\boldsymbol{e}) = \boldsymbol{\omega}^t\boldsymbol{e} + \varepsilon, \tag{1.2}$$

where $\boldsymbol{\omega} = (\omega_1, ..., \omega_n)^t \in \mathbb{R}^{n+}$ is the vector of performance measure sensitivities. The random component ε is normally distributed with zero mean and variance σ^2, and represents potential effects on the performance measure beyond the agent's control.

As pointed out by Feltham and Xie [1994], the performance measure does not necessarily capture the agent's contribution to the gross payoff perfectly. Formally, if there exists a constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda\boldsymbol{\omega}$, performance measure $P(\boldsymbol{e})$ is congruent with the gross payoff $V(\boldsymbol{e})$.[8] Otherwise, the performance measure is incongruent and its application in an incentive contract motivates the agent to implement an inefficient effort allocation across tasks [Feltham and Xie, 1994, Baker, 2002].

Baker [2002] provided a geometric measure for performance measure congruity. Since his result is fundamental to the subsequent analysis, it is summarized in the following definition.

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[7] A similar approach is used by MacLeod [1996], where ψ_i, $i = 1, ..., n$, are random variables. However, his work is different in the sense that he focuses on the relationship between explicit and implicit incentive contracts rather than on the effort distortion induced by incongruent performance measurement.

[8] This phenomenon is described by several terms in the multi-task agency literature: *performance measure congruity* [Feltham and Xie, 1994, Bushman et al., 2000, Hughes et al., 2005], *non-distorted performance measure* [Baker, 2000, 2002], and *goal congruence* [Anthony and Govindarajan, 1995, Banker and Thevaranjan, 2000]. For the sake of consistence, I use the term *performance measure congruity* throughout this essay.

Definition 1.1 *The congruence of performance measure $P(\boldsymbol{e})$ to gross payoff $V(\boldsymbol{e})$ with respect to the marginal effect of $\boldsymbol{e}$ is measured by $\Upsilon^C(\varphi) = \cos\varphi$, where φ is the angle between the vector of gross payoff sensitivities $\boldsymbol{\mu}$ and the vector of performance measure sensitivities $\boldsymbol{\omega}$.*

Accordingly, as long as vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\omega}$ are linearly independent, the performance measure does not reflect the agent's contribution to gross payoff, and therefore, is incongruent. Formally, there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda\boldsymbol{\omega}$, thereby implying $\varphi \neq 0$. A more congruent performance measure thereby implies a smaller angle φ and leads to a higher measure of congruity $\Upsilon^C(\varphi)$ due to the definition of the cosine. Finally note that $\varphi \in [0, \pi/2]$ since $\mu_i, \omega_i \geq 0$, $i = 1, ..., n$, where φ is represented in radian measure.

In line with previous multi-task literature, I restrict my analysis to a compensation scheme w which is linear in performance measure $P(\boldsymbol{e})$. The payment w takes therefore the form

$$w(\boldsymbol{e}) = \alpha + \beta P(\boldsymbol{e}), \tag{1.3}$$

where α denotes the fixed payment and β denotes the incentive parameter. The transfer α is utilized to split the surplus between the principal and the agent, whereas β is used to provide the agent with incentives for implementing effort.

Remark As shown by Holmström [1979] and Grossman and Hart [1983], optimal contracts are not necessarily linear.[9] However, linear contracts are widely used in economic analyzes, particulary in the multi-task agency literature, see e.g. Feltham and Xie [1994], Bushman et al. [2000], Banker and Thevaranjan [2000], Baker [2002], and Hughes et al. [2005]. The advantage of using linear compensation schemes in multi-task agency models is that they provide additional insights in the agent's choice for allocating her effort across tasks. Nevertheless, for a continuous time model with Brownian motion where the agent controls the drift rate, Holmström and Milgrom [1987] found the optimal incentive scheme to be linear in output.[10] Thus, we can interpret a single-period framework as a one-shot consideration of the continuous time model proposed by Holmström and Milgrom [1987]. However, their result of linear contracts being optimal applies only to settings with a single performance measure and is not valid for settings with multiple measures as considered in section 1.7. For a general characterization of optimal contracts in a multi-task agency framework refer to Gjesdal [1982].

Since the compensation scheme is linear, the agent's utility is exponential, and the error term is normally distributed, maximizing the agent's expected utility is analogous to maximizing her certainty equivalent

$$CE(\boldsymbol{e}) = \alpha + \beta\boldsymbol{\omega}^t\boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t\boldsymbol{\Psi}\boldsymbol{e} - \frac{\rho}{2}\beta^2\sigma^2, \tag{1.4}$$

where $\rho\beta^2\sigma^2/2$ is the required risk premium in order to compensate the agent for the uncertainty in her incentive payment $\beta P(\boldsymbol{e})$.

[9] For the analysis of the shape of optimal contracts refer to Hemmer et al. [2000].

[10] See as well Hellwig and Schmidt [2002] for the approximation of the continuous time model by a discrete-time model where optimal incentive schemes are also linear.

The timing of this problem is as follows. First, the principal offers the agent a contract (α^*, β^*). If this contract guarantees the agent at least the same expected utility as her best alternative, she accepts. After the agent implemented $\boldsymbol{e}$ and the random variables ε and ε_V are realized, the payments take place.

For clarification, I subsequently illustrate the distinction between effort intensity and effort allocation. Formally, let two arbitrary activities e_k and e_j vary to $\hat{e}_k$ and $\hat{e}_j$, respectively. If the ratio between both activities remains identical such that $e_k/e_j = \hat{e}_k/\hat{e}_j$, $k, j = 1, ..., n$, $k \neq j$, the relative effort allocation remains the same. In contrast, if $e_k/e_j \neq \hat{e}_k/\hat{e}_j$ for at least one pair $(k, j) \in \{1, ..., n\}$, $k \neq j$, the relative effort allocation varies. The overall effort intensity, however, changes without affecting the effort allocation, if there exists a constant $\lambda > 0$ satisfying $\boldsymbol{e} = \lambda\hat{\boldsymbol{e}}$, where $\hat{\boldsymbol{e}}$ is the modified effort vector.

For the ease of comparing different effort allocations, it is useful to commit to the subsequent definition throughout this essay.

Definition 1.2 *The agent implements a distorted effort allocation if there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda\boldsymbol{e}$.*

The implemented effort allocation is referred to be distorted if it does not reflect the agent's marginal contribution to gross payoff $V(\boldsymbol{e})$. Note, however, that non-distortion is not necessarily optimal since this concept does not incorporate the corresponding costs for implementing an arbitrary effort vector.

1.3 The First-Best Contract

Before I move on to the second-best contract, it is useful to derive the first-best solution of this problem as a benchmark for the subsequent analyzes. Then, the first-best effort allocation and intensity can be compared to the second-best environment, where the agent's effort is non-contractible so that moral hazard occurs.

Suppose the principal can specify a desired effort intensity and allocation in an enforceable contract. In this case, she appoints the effort vector $\boldsymbol{e}$ which maximizes the difference between the expected gross payoff $V(\boldsymbol{e})$ and costs $w = C(\boldsymbol{e})$:

$$\max_{\boldsymbol{e}} \ \Pi(\boldsymbol{e}) = \boldsymbol{\mu}^t\boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t\boldsymbol{\Psi}\boldsymbol{e}. \tag{1.5}$$

Let $\boldsymbol{\phi} \equiv \boldsymbol{\Psi}^{-1}\boldsymbol{\mu} = (\mu_1/\psi_1, ..., \mu_n/\psi_n)^t$ be the vector of the payoff-cost sensitivity ratios. Then, the first-best effort vector is

$$\boldsymbol{e}^{fb} = \boldsymbol{\phi}. \tag{1.6}$$

The principal maximizes her expected profit by assigning each activity e_i in accordance to its payoff-cost sensitivity ratio μ_i/ψ_i, $i = 1, ..., n$. Activities with high ratios are consequently more intensively conducted relative to activities with low ratios.

Recall that $\boldsymbol{e}^{fb}$ is distorted if there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda\boldsymbol{e}^{fb}$, see definition 1.2. This implies that first-best effort is non-distorted if $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, i.e. the agent has no comparative advantage in performing some tasks relative to others. In contrast, if the agent has different abilities across tasks, it is optimal to implement a distorted effort allocation in order to balance the benefits and costs of all relevant tasks.

By substituting $\boldsymbol{e}^{fb}$ in (1.5) and using the relation $\boldsymbol{\mu}^t\boldsymbol{\phi} = \|\boldsymbol{\mu}\|\|\boldsymbol{\phi}\|\cos\kappa$ for vector products, the expected first-best profit becomes

$$\Pi^{fb} = \frac{1}{2}\|\boldsymbol{\mu}\|\|\boldsymbol{\phi}\|\cos\kappa, \tag{1.7}$$

where κ is the angle between vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\phi}$, and $\|\cdot\|$ denotes the length of the respective vector.

The agent's task-specific abilities affect the expected first-best profit in two ways. The first effect is a result of the overall cost intensity for implementing an arbitrary effort vector. To illustrate this effect, consider two agents A and B characterized by $\boldsymbol{\Psi}^A$ and $\boldsymbol{\Psi}^B$, respectively. If $\boldsymbol{\Psi}^A = \lambda\boldsymbol{\Psi}^B$, $\lambda > 1$, agent A exhibits a less overall cost intensity than agent B for the implementation of an arbitrary effort vector. Observe, however, that both agents share the same relative task-specific abilities across tasks. Therefore, $\lambda\|\boldsymbol{\phi}^A\| = \|\boldsymbol{\phi}^B\|$, whereas $\kappa^A = \kappa^B$. The second effect follows from the relation between the payoff sensitivities $\boldsymbol{\mu}$ and the agent's relative task-specific abilities $\boldsymbol{\Psi}$. Consider for instance the agent's ability ψ_i to perform task i. If this ability is increasing (i.e. ψ_i decreases) relative to the other abilities, the agent could implement the same effort vector, but suffers less disutility of effort for performing task i. In this case, $\|\boldsymbol{\phi}\|$ increases. However, the effect on κ is ambiguous. Particularly, decreasing ψ_i leads to a higher angle κ if $\psi_i < 1$, and to a lower κ, otherwise. For the principal, however, it is optimal to enhance e_i^{fb} until the marginal benefit of task i is equal to its marginal costs, i.e. $\mu_i = \psi_i e_i$. Consequently, Π^{fb} increases. This eventually implies that a potential decline in $\cos\kappa$ is preponderated by an increase of $\|\boldsymbol{\phi}\|$.

1.4 The Second-Best Contract

If the principal cannot directly contract over $\boldsymbol{e}$, she faces an incentive problem for motivating the agent to implement appropriate effort. Since the gross payoff $V(\boldsymbol{e})$ is non-verifiable, the only contractible information is the performance measure $P(\boldsymbol{e})$. However, the application of $P(\boldsymbol{e})$ in an incentive contract may cause two inefficiencies. First, the performance measure—and therefore the agent's compensation—is uncertain such that the risk-averse agent requires a risk premium for accepting a contract dependent on $P(\boldsymbol{e})$. Second, the performance measure can be incongruent and, therefore, motivate the agent to inefficiently allocate her effort across tasks. The subsequent analysis focuses on the second inefficiency since the trade-off between incentive risk and the agent's desire for insurance is intensively analyzed by previous literature.[11]

In a second-best environment, the principal's problem is to design a contract (α^*, β^*) that maximizes her expected profit $\Pi = \mathrm{E}[V(\boldsymbol{e}) - w(\boldsymbol{e})]$ while ensuring the

[11]For a detailed analysis in a LEN-setting, see e.g. Spremann [1987], Baker [1992], and Prendergast [1999]; and for a general approach Shavell [1979], Holmström [1979], Grossman and Hart [1983], and Rees [1985].

agent's participation. The optimal linear contract therefore solves

$$\max_{\alpha,\beta,\boldsymbol{e}} \ \Pi \equiv \boldsymbol{\mu}^t\boldsymbol{e} - \alpha - \beta\boldsymbol{\omega}^t\boldsymbol{e} \tag{1.8}$$

s.t.

$$\boldsymbol{e} = \arg\max_{\tilde{\boldsymbol{e}}} \alpha + \beta\boldsymbol{\omega}^t\tilde{\boldsymbol{e}} - \frac{1}{2}\tilde{\boldsymbol{e}}^t\boldsymbol{\Psi}\tilde{\boldsymbol{e}} - \frac{\rho}{2}\beta^2\sigma^2 \tag{1.9}$$

$$\alpha + \beta\boldsymbol{\omega}^t\boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t\boldsymbol{\Psi}\boldsymbol{e} - \frac{\rho}{2}\beta^2\sigma^2 \geq 0, \tag{1.10}$$

where (1.9) is the agent's incentive condition and (1.10) her participation constraint.

First, observe that (1.9) can be replaced by $\boldsymbol{e} = \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}\beta$. For the subsequent analysis, let $\boldsymbol{\Gamma} \equiv \boldsymbol{\Psi}^{-1}\boldsymbol{\omega} = (\omega_1/\psi_1, ..., \omega_n/\psi_n)^t$ be the vector of measure-cost sensitivity ratios. Thus, the agent implements

$$\boldsymbol{e}^* = \boldsymbol{\Gamma}\beta. \tag{1.11}$$

In contrast to the first-best scenario, the agent's effort e_i for performing task i depends on the measure-cost sensitivity ratio ω_i/ψ_i and the incentive parameter β.

In order to maximize her expected profit, the principal sets α such that the agent's participation constraint is binding. By solving (1.10) for α and substituting the resulting expression together with $\boldsymbol{e}^*$ in the principal's objective function (1.8), the maximization problem simplifies to

$$\max_{\beta} \ \Pi \equiv \boldsymbol{\mu}^t\boldsymbol{\Gamma}\beta - \frac{\beta^2}{2}\left[\boldsymbol{\omega}^t\boldsymbol{\Gamma} + \rho\sigma^2\right]. \tag{1.12}$$

The first-derivative of Π with respect to β gives the optimal incentive parameter

$$\beta^* = \frac{\boldsymbol{\mu}^t\boldsymbol{\Gamma}}{\boldsymbol{\omega}^t\boldsymbol{\Gamma} + \rho\sigma^2}. \tag{1.13}$$

Besides the precision of the performance measure, $1/\sigma^2$, with the agent's risk tolerance, $1/\rho$, the optimal incentive parameter is a function of the gross payoff sensitivities $\boldsymbol{\mu}$, the performance measure sensitivities $\boldsymbol{\omega}$, and the measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. Recall that $\boldsymbol{\Gamma} = \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}$, i.e. $\boldsymbol{\Gamma}$ comprises the agent's task-specific abilities $\boldsymbol{\Psi}$. Hence, β^* incorporates $\boldsymbol{\Psi}$ in two ways: (i) by its relation to the gross payoff sensitivities $\boldsymbol{\mu}$ in the numerator; and (ii), by its relation to the performance measure sensitivities $\boldsymbol{\omega}$ in the numerator and denominator. It can therefore be inferred that agents with different task-specific abilities may obtain diverse incentive contracts, even if they are in charge of performing an identical set of tasks and evaluated by the same information system.

Substituting β^* in (1.12) and using geometric representations give the principal's expected second-best profit

$$\Pi^* = \frac{\|\boldsymbol{\mu}\|^2\|\boldsymbol{\Gamma}\|^2\cos^2\theta}{2(\|\boldsymbol{\omega}\|\|\boldsymbol{\Gamma}\|\cos\xi + \rho\sigma^2)}, \tag{1.14}$$

where θ denotes the angle between the vector of payoff sensitivities $\boldsymbol{\mu}$ and the vector of measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. The angle between the vector of performance measure sensitivities $\boldsymbol{\omega}$ and vector $\boldsymbol{\Gamma}$ is denoted by ξ.

1.5 Performance Measure Congruity and Effort Distortion

In this section, I focus more intensively on performance measure congruity and its effect on effort distortion if the agent performs different tasks with varying degrees of ease.

Performance measure congruity refers to the degree of alignment between the agent's marginal effect on her performance measure and on the expected gross payoff [Feltham and Xie, 1994]. Performance measure congruity can thus be characterized by the angle φ between the vector of payoff sensitivities $\boldsymbol{\mu}$ and the vector of performance measure sensitivities $\boldsymbol{\omega}$, as emphasized by Baker [2002]. In contrast, effort distortion refers to the relation between an implemented effort vector $\boldsymbol{e}$ and the vector of the payoff sensitivities $\boldsymbol{\mu}$. If the agent's effort allocation reflects its relative contribution to $V(\boldsymbol{e})$, her effort is non-distorted, see definition 1.2. However, as shown in section 1.3, effort distortion is not necessarily inefficient. Even the first-best effort is distorted if the agent has comparative advantages in performing some tasks relative to others. Nevertheless, a distorted effort allocation is inefficient if it deviates from the one implemented under first-best. The agent implements an efficient (first-best) effort allocation if there exists a constant $\lambda > 0$ satisfying $\boldsymbol{e}^{fb} = \lambda \boldsymbol{e}^*$. Recall that $\boldsymbol{e}^{fb} = \boldsymbol{\Psi}^{-1}\boldsymbol{\mu}$ and $\boldsymbol{e}^* = \beta\boldsymbol{\Psi}^{-1}\boldsymbol{\omega}$. This leads to the first observation.

Corollary 1.1 *Only a congruent performance measure with $\boldsymbol{\mu} = \lambda\boldsymbol{\omega}$, $\lambda \in \mathbb{R}^*$, leads to a first-best effort allocation. If in addition $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, the second-best effort vector $\boldsymbol{e}^*$ is non-distorted.*

Observe that the first part of this corollary is independent of the agent's task-specific abilities. Consequently, I achieve the same observation as Feltham and Xie [1994], even though I incorporate task-specific abilities. If the applied performance measure is incongruent, we can infer that the agent is motivated to implement an inefficient effort allocation, regardless of her characteristics. However, the extent of this inefficiency is determined by $\boldsymbol{\Psi}$. Finally, identical task-specific abilities additionally lead to non-distorted effort if the applied performance measure is congruent. The rationale for this observation is that identical abilities for performing all relevant tasks imply that the agent's preference for her effort allocation is only determined by the relative contribution of her tasks to the performance measure. If this measure reflects the agent's relative contribution to firm value, i.e. it is congruent, she is motivated to implement non-distorted effort.

As explained earlier, the agent's second-best effort can deviate from that implemented under first-best in two dimensions. First, the agent can choose a suboptimal effort intensity over all tasks; and second, she may implement an inefficient effort allocation across tasks. Therefore, an important question is, how the principal can regulate both dimensions in a second-best environment, provided the agent exhibits task-specific abilities.

Proposition 1.1 *The principal can adjust the agent's effort intensity by varying the incentive parameter β. In contrast, if only one performance measure is available, the effort allocation is exogenously determined by $\boldsymbol{\Gamma}$.*

Proof See appendix.

By providing an appropriate incentive parameter β, the principal can motivate the agent to implement any desired effort intensity. The optimal incentive parameter—as well known—thereby trades-off the provision of incentives and the agent's desire for insurance. In contrast to the effort intensity, the effort allocation cannot be controlled by the principal, as long as the underlying information system generates only one performance measure. However, I show in section 1.7 that the principal can adjust the effort allocation when she receives multiple performance measures.

It can be deduced from previous observations that $\mathbf{\Gamma}$ plays an important role for the induced effort allocation. For the subsequent analysis, suppose that for at least two activities e_k and e_j the respective abilities are not identical. Formally, $\psi_k \neq \psi_j$ for at least one pair $(k, j) \in \{1, ..., n\}$, $k \neq j$. Consequently, vector $\mathbf{\Gamma}$ is linearly independent of vector $\boldsymbol{\omega}$.

Proposition 1.2 *If $\psi_k \neq \psi_j$ for at least one pair $(k, j) \in \{1, ..., n\}$, $k \neq j$, then $\Upsilon^D(\theta) = \cos\theta$ measures effort distortion under second-best.*

Proof See appendix.

Note that the measure $\Upsilon^D(\theta)$ is negatively related to effort distortion. The less distorted the agent's effort allocation with respect to $\boldsymbol{\mu}$ is, the smaller is θ, and consequently, the higher is $\Upsilon^D(\theta)$. If $\theta = 0$, the application of performance measure $P(\boldsymbol{e})$ motivates non-distorted effort. Observe, however, that an incongruent performance measure induces non-distorted effort if $\boldsymbol{\mu} = \lambda\beta\mathbf{\Gamma}$, $\lambda \in \mathbb{R}^*$, or equivalently,

$$\boldsymbol{\omega} = \mathbf{\Psi}\boldsymbol{\mu}\,(\lambda\beta)^{-1}\,. \tag{1.15}$$

In this case, the performance measure sensitivities $\boldsymbol{\omega}$ are a transformation of the agent's marginal contribution to gross payoff $\boldsymbol{\mu}$ and her task-specific abilities $\mathbf{\Psi}$. However, as pointed out by corollary 1.1, a non-distorted effort allocation can only be optimal if $P(\boldsymbol{e})$ is perfectly congruent and the agent experiences identical abilities for performing all relevant tasks.

Suppose the available performance measure $P(\boldsymbol{e})$ changes such that the agent is motivated to implement a less distorted effort allocation. Formally, θ decreases. This implies, *ceteris paribus*, a higher expected profit Π^*. Note, however, that there is a second effect on Π^* captured by ξ as the angle between $\boldsymbol{\omega}$ and $\mathbf{\Gamma}$. To illustrate this effect, we can re-formulate the agent's effort costs by substituting $\boldsymbol{e}^*$:

$$C(\cdot) = \frac{1}{2}\beta^2\|\boldsymbol{\omega}\|\|\mathbf{\Gamma}\|\cos\xi. \tag{1.16}$$

The properties of the agent's task-specific abilities affect her effort costs in two ways. The first effect is a result of the effort cost intensity over all tasks. For illustrative purposes, assume that the effort costs take the form $C(\boldsymbol{e}) = \boldsymbol{e}^t\lambda\mathbf{\Psi}\boldsymbol{e}/2$ with $\lambda > 0$. Increasing λ implies that all tasks become more costly to perform, thereby leading to a higher $\|\mathbf{\Gamma}\|$ without affecting $\cos\xi$. The second effect is caused by the relation between the performance measure sensitivities $\boldsymbol{\omega}$ and the agent's task-specific abilities $\mathbf{\Psi}$. The relative abilities across tasks thereby affect $\|\mathbf{\Gamma}\|$ and

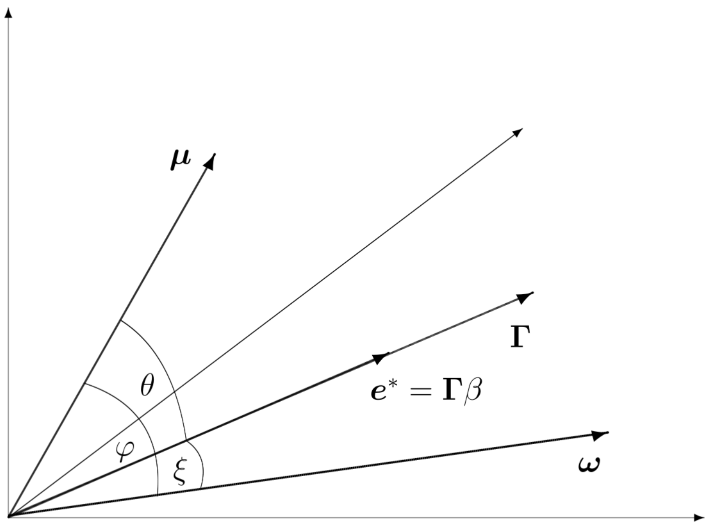

Figure 1.1: Performance Measure Congruity and Effort Distortion for $n = 3$

$\cos\xi$. Recall that $\|\boldsymbol{\Gamma}\|$ determines the effort intensity without affecting the allocation. In contrast, $\cos\xi$ measures the agent's effort costs (in utility terms) for a particular effort allocation motivated by $P(\boldsymbol{e})$.

Corollary 1.2 *If $\psi_k \neq \psi_j$ for at least one pair $(k, j) \in \{1, ..., n\}$, $k \neq j$, then $\Upsilon^{M/C}(\xi) = \cos\xi$ characterizes the measure-cost efficiency.*

The previous results are illustrated in figure 1.1 for the three-dimensional case ($n = 3$). Besides the second-best effort vector $\boldsymbol{e}^*$, it depicts the vectors of the gross payoff sensitivities $\boldsymbol{\mu}$, performance measure sensitivities $\boldsymbol{\omega}$, and measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. The effort vector $\boldsymbol{e}^*$ has the same direction as $\boldsymbol{\Gamma}$, only their lengths differ, depending on β. Observe that $\boldsymbol{e}^*$ is not necessarily on the plane spanned by $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$. The location of $\boldsymbol{e}^*$ relative to $\boldsymbol{\mu}$ characterizes the induced effort distortion (angle θ), whereas the relation between $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$ measures the congruity of performance measure $P(\boldsymbol{e})$ (angle φ). Finally, the measure-cost efficiency is characterized by the relation of $\boldsymbol{\Gamma}$ to $\boldsymbol{\omega}$ (angle ξ).

If vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\omega}$ point in the same direction, then $\boldsymbol{e}^{fb} = \lambda\boldsymbol{e}^*$, $\lambda > 0$, i.e. the incentive contract motivates the agent to implement the first-best effort allocation, see corollary 1.1. Nevertheless, inducing a first-best effort intensity by adjusting β can only be optimal if the agent is either risk-neutral or the performance measure is perfectly precise. Otherwise, the principal imposes to much incentive risk on the agent which requires the payment of a higher risk premium to ensure her participation.

Now consider the case where the agent has identical abilities for all tasks, i.e. $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. As a consequence, $\boldsymbol{\Gamma} = \boldsymbol{\omega}/\hat{\psi}$ so that vector $\boldsymbol{\Gamma}$ and vector $\boldsymbol{\omega}$ point in the same direction. This additionally implies that $\boldsymbol{e}^* = \boldsymbol{\omega}\beta/\hat{\psi}$ and $\xi = 0$. Thus, $\boldsymbol{e}^*$ and $\boldsymbol{\omega}$ are identical with respect to their direction, only their lengths differ, depending on β and $\hat{\psi}$. Accordingly, the measure of congruity is now identical to the measure of distortion. This observation is summarized and proofed by the subsequent proposition.

Proposition 1.3 *If* $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, *then* $\Upsilon^D(\varphi) = \Upsilon^C(\varphi) = \cos\varphi$.

Proof See appendix.

If agents do not exhibit different task-specific abilities, performance measure congruity and effort distortion are captured by the same measure. However, if we allow the agent to possess different abilities across tasks, it becomes pivotal to distinguish between both concepts. The application of incongruent performance measures in incentive contracts leads to inefficient effort allocations, but the extent of these inefficiencies are further determined by the agent's relative abilities for performing the relevant tasks.

Consider again the expected second-best profit Π^* from section 1.4. According to the previous observations, it depends on three elements: (i) the measure of distortion $\Upsilon^D(\theta)$ in the numerator; (ii) the measure-cost efficiency $\Upsilon^{M/C}(\xi)$ in the denominator; and (iii), the agent's risk aversion ρ in conjunction with the variance σ^2 of the applied performance measure in the denominator. It is common knowledge that the trade-off between incentive risk and the agent's desire for insurance affects optimal incentive contracts. Moreover, as demonstrated by Feltham and Xie [1994] and Baker [2002], incentive contracts in multi-task agency relations are adjusted to the congruity of applied performance measures. However, the previous analysis indicates that the measure-costs efficiency is a third crucial factor whenever the agent performs some tasks more efficiently than others due to task-specific abilities.

1.6 Ranking Performance Measures

As Feltham and Xie [1994] emphasized, performance measures may differ with respect to their congruity and precision. The previous analysis additionally indicates that task-specific abilities play a crucial role for the contract efficiency. This section therefore focuses on how the attributes of performance measures and agents eventually determine the relative value of measures in multi-task agencies.

Consider a set $\mathbf{P}$ of $m \geq 2$ performance measures $P_i(\boldsymbol{e}) = \boldsymbol{\omega}_i^t \boldsymbol{e} + \varepsilon_i$, with $P_i(\boldsymbol{e}) \in \mathbf{P} \subseteq \mathbb{R}^m$ and $\varepsilon_i \sim N(0, \sigma_i^2)$.[12] To illustrate the relative value of individual performance measures, we can compare the expected profits each of them would induce if applied in the agent's incentive contract. Then, performance measure $P_k(\boldsymbol{e})$ is referred to be strictly superior, if it provides the principal a strictly higher expected profit than all other available measures $P_i(\boldsymbol{e}) \in \mathbf{P}$, $i \neq k$. Thus, I first ignore the value of combining several measures and defer the consideration of this possibility to the next section.

For single-task agency relations, Kim and Suh [1991] have shown that the value of performance measures can be compared by their respective signal/noise ratio. By adjusting their definition to a multi-task agency setting, the signal/noise ratio of performance measures $P_i(\boldsymbol{e})$ is

$$\Lambda_i = \frac{(\nabla P_i(\boldsymbol{e}^*))^t \, (\nabla P_i(\boldsymbol{e}^*))}{\sigma_i^2}, \tag{1.17}$$

[12] Subscript i refers henceforth to performance measure $P_i(\boldsymbol{e}) \in \mathbf{P}$.

where $\nabla P_i(\boldsymbol{e}^*)$ is the gradient of performance measure $P_i(\boldsymbol{e})$ with respect to $\boldsymbol{e}$. In single-task agencies, performance measures with higher signal/noise ratios provide more precise information about the implemented effort and are therefore preferred to measures with lower ratios. In this multi-task setting, the signal/noise ratio of performance measures $P_i(\boldsymbol{e})$ is

$$\Lambda_i = \frac{\|\boldsymbol{\omega}_i\|^2}{\sigma_i^2}. \tag{1.18}$$

One can infer from the previous analysis that signal/noise ratios are not necessarily sufficient to rank performance measures in multi-task agencies, especially, when agents differ in their task-specific abilities. This deduction is supported by the next proposition.

Proposition 1.4 *Performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if,*

$$\frac{\|\boldsymbol{\omega}_k\|}{\|\boldsymbol{\Gamma}_k\|}\frac{\Upsilon^{M/C}(\xi_k)}{(\Upsilon^D(\theta_k))^2} + \frac{\rho\sigma_k^2}{\|\boldsymbol{\Gamma}_k\|^2(\Upsilon^D(\theta_k))^2} < \frac{\|\boldsymbol{\omega}_j\|}{\|\boldsymbol{\Gamma}_j\|}\frac{\Upsilon^{M/C}(\xi_j)}{(\Upsilon^D(\theta_j))^2} + \frac{\rho\sigma_j^2}{\|\boldsymbol{\Gamma}_j\|^2(\Upsilon^D(\theta_j))^2}, \tag{1.19}$$

where $\Upsilon^D(\theta_i)$ is the measure of distortion induced by $P_i(\boldsymbol{e})$, and $\Upsilon^{M/C}(\xi_i)$ is the related quantification for the measure-cost efficiency, $i = \{j, k\}$.

Proof Follows directly by rearranging $\Pi^*(P_k(\boldsymbol{e})) > \Pi^*(P_j(\boldsymbol{e}))$ and substituting $\Upsilon^{M/C}(\xi_i) = \cos\xi_i$ and $\Upsilon^D(\theta_i) = \cos\theta_i$, $i = k, j$.

The value of a performance measure in comparison to any other measure is contingent on two ratios: (i) the normalized ratio between the measure-cost efficiency $\Upsilon^{M/C}(\cdot)$ and the induced effort distortion $\Upsilon^D(\cdot)$; and, (ii) the normalized inverse of the distortion measure $\Upsilon^D(\cdot)$ with the precision $1/\sigma_k^2$ of the performance measure and the agent's risk tolerance $1/\rho$. Observe finally that performance measure congruity does not directly enter into this ranking criteria. It, however, affects indirectly the measure of effort distortion $\Upsilon^D(\theta_i)$ and the measure-cost efficiency characterized by $\Upsilon^{M/C}(\xi_i)$.

In fact, the value of performance measures in multi-task agencies cannot necessarily be compared by their respective signal/noise ratios. It is rather pivotal to take the induced effort distortion and measure-cost efficiency into consideration—both determined by the performance measure sensitivities $\boldsymbol{\omega}_i$ relative to the agent's task specific abilities $\boldsymbol{\Psi}$. Therefore, comparing the value of performance measures requires specific knowledge about the agent's characteristics, which is not necessary for ranking performance measures in single-task agencies. In multi-task agencies, however, the agent's characteristics eventually determine the principal's preference for a specific information system.

Corollary 1.3 *Suppose $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. Then, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if,*

$$\frac{1}{\Upsilon^C(\varphi_k)}\left[1 + \hat{\psi}\rho\Lambda_k^{-1}\right]^{\frac{1}{2}} < \frac{1}{\Upsilon^C(\varphi_j)}\left[1 + \hat{\psi}\rho\Lambda_j^{-1}\right]^{\frac{1}{2}}, \tag{1.20}$$

where Λ_i, $i = \{j, k\}$, is the signal/noise ratio of performance measure $P_i(\boldsymbol{e})$, and $\Upsilon^C(\varphi_i)$ its congruity measure.

Proof See appendix.

If the agent's preference for an effort allocation depends only on the characteristics of her performance evaluation since her abilities are identical for all tasks, we can use adjusted signal/noise ratios to rank performance measures in multi-task agencies. Nevertheless, it is still required to know $\hat{\psi}$ and ρ in order to assess the relative value of performance measures.

The subsequent proposition offers a sufficient condition ensuring that performance measures can be ranked exclusively by their respective signal/noise ratios, and therefore, independent of the agent's characteristics.

Proposition 1.5 *Suppose there exist constants $\lambda_j \neq 0$ satisfying $\boldsymbol{\omega}_i = \lambda_j \boldsymbol{\omega}_j$ for all $i, j = 1, ..., m$, $i \neq j$. Then, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if, $\Lambda_k > \Lambda_j$.*

Proof See appendix.

Accordingly, the signal/noise ratio is sufficient to rank performance measures in multi-task agencies, if all measures provide the same information about the agent's relative effort allocation. In this case, observe that $\Upsilon^C(\varphi_i) = \Upsilon^C(\varphi_j)$, $i, j = 1, ..., m$, i.e. all performance measures share the same measure of congruity.[13] As a consequence, every available performance measure—if applied in the agent's incentive contract—would imply the same effort distortion and measure-cost efficiency. Then, their relative value is defined by their precision and scale, which in turn is represented by their respective signal/noise ratio.

To investigate the effects of task-specific abilities on the ordering of performance measures, it is insightful to eliminate effects related to their precision. By setting $\rho = 0$, condition (1.19) simplifies to

$$\nu \, \frac{\cos^2 \theta_k}{\cos^2 \theta_j} > \frac{\cos \xi_k}{\cos \xi_j}, \qquad \nu = \frac{\|\boldsymbol{\omega}_j\|}{\|\boldsymbol{\omega}_k\|} \frac{\|\boldsymbol{\Gamma}_k\|}{\|\boldsymbol{\Gamma}_j\|}. \tag{1.21}$$

The value of performance measure $P_k(\boldsymbol{e})$ relative to $P_j(\boldsymbol{e})$ depends—besides on their precision and scaling as previously emphasized—on their relative effort distortion ($\cos \theta_i$) and relative measure-cost efficiency ($\cos \xi_i$) weighted by the multiplier ν, $i = k, j$. In order to make both measures comparable, it is essential to normalize their scale $\|\boldsymbol{\omega}_i\|$, and exclude their effect on $\|\boldsymbol{\Gamma}_i\|$, $i = k, j$. Accordingly, if either the agent is risk-neutral or the realization of performance measures is not influenced by random effects, the relative value of performance measures depends on two factors: (i) the motivated effort allocation and its contribution to gross payoff $V(\boldsymbol{e})$; and, (ii) the imposed costs to motivate this effort allocation.

1.7 Multiple Performance Measures

Even though the consideration of single performance measures provides important insights into incentive mechanisms when agents are placed in charge of several tasks,

[13] Note that the reversed inference cannot be made, i.e. if $\Upsilon^C(\varphi_i) = \Upsilon^C(\varphi_j)$, it is not necessarily true that $\boldsymbol{\omega}_i = \lambda_j \boldsymbol{\omega}_j$, $\lambda_j \neq 0$, $i, j = 1, ..., m$, $i \neq j$. In this case, the signal/noise ratio is not sufficient to rank performance measures in multi-task agencies.

it is more reasonable to assume that the principal has access to multiple performance measures, e.g. different accounting numbers. If these additional measures are informative, they should be used to improve incentive contracts [Holmström, 1979]. This section focuses on the optimal aggregation of multiple performance measures, when the agent exhibits different task-specific abilities.

For the subsequent analysis, suppose an information system generates an m-dimensional vector of performance measures $\boldsymbol{P} = (P_1(\boldsymbol{e}), ..., P_m(\boldsymbol{e}))^t$, $\boldsymbol{P} \in \mathbb{R}^m$. Let $\Xi = (\boldsymbol{\omega}_1^t, ..., \boldsymbol{\omega}_m^t)^t$ be the $m \times n$ matrix of the respective performance measure sensitivities, where the n-dimensional vector $\boldsymbol{\omega}_i$ summarizes the performance measure sensitivities of $P_i(\boldsymbol{e})$. Accordingly, $\boldsymbol{P}$ can be written as

$$\boldsymbol{P} = \Xi \boldsymbol{e} + \boldsymbol{\varepsilon}, \tag{1.22}$$

where $\boldsymbol{\varepsilon} = (\varepsilon_1, ..., \varepsilon_m)^t$ is a normally distributed m-dimensional vector of random variables with zero mean and covariance matrix $\boldsymbol{\Sigma}$. Due to the more general characteristic of the subsequent analysis, we can now relax our initial assumption with respect to $\boldsymbol{\Psi}$ and may assume that some elements in $\boldsymbol{\Psi}$ beyond the diagonal are strictly positive or strictly negative, i.e. some activities are complements or substitutes. In order to ensure that its inverse exists, $\boldsymbol{\Psi}$ is assumed to be a positive definite matrix.

If the principal applies multiple performance measures in the agent's incentive contract, her certainty equivalent modifies to

$$CE(\boldsymbol{e}) = \alpha + \boldsymbol{\beta}^t \Xi \boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t \boldsymbol{\Psi} \boldsymbol{e} - \frac{\rho}{2}\boldsymbol{\beta}^t \boldsymbol{\Sigma} \boldsymbol{\beta}, \tag{1.23}$$

where $\boldsymbol{\beta} = (\beta_1, ..., \beta_m)^t$ is an m-dimensional vector of incentive parameters and represents the weight for each performance measure in the linear aggregation. Since the noise terms are normally distributed, the linear aggregation of performance measures is optimal [Banker and Datar, 1989].

The solution concept for deducing the optimal linear contract dependent on $\boldsymbol{P}$ is similar to the one applied in section 1.4. First, the agent maximizes her certainty equivalent by choosing

$$\boldsymbol{e}^* = \boldsymbol{\Psi}^{-1} \Xi^t \boldsymbol{\beta}. \tag{1.24}$$

The agent's preference for an effort allocation depends on her task-specific abilities $\boldsymbol{\Psi}$ and the marginal effect of each task on her aggregated performance evaluation $\Xi^t \boldsymbol{\beta}$. In contrast to the single performance measure case, the principal can now influence the agent's effort allocation by adjusting the weight β_i, thereby altering the agent's marginal effect on her performance evaluation.

The principal's problem is to define a contract $(\alpha^*, \boldsymbol{\beta}^*)$, dependent on $\boldsymbol{P}$, which maximizes her expected profit $\Pi = \mathrm{E}[V(\boldsymbol{e}) - w(\boldsymbol{e})]$. In order to minimize costs, it is optimal to set α such that the agent's participation constraint is binding. Solving $CE(\boldsymbol{e}) = 0$ for α and substituting this expression together with $\boldsymbol{e}^* = \boldsymbol{\Psi}^{-1} \Xi^t \boldsymbol{\beta}$ in the principal's objective function yield an unconstrained maximization problem:

$$\max_{\boldsymbol{\beta}} \Pi \equiv \boldsymbol{\mu}^t \boldsymbol{\Psi}^{-1} \Xi^t \boldsymbol{\beta} - \frac{1}{2}\boldsymbol{\beta}^t \Xi \boldsymbol{\Psi}^{-1} \Xi^t \boldsymbol{\beta} - \frac{\rho}{2}\boldsymbol{\beta}^t \boldsymbol{\Sigma} \boldsymbol{\beta}. \tag{1.25}$$

The first-order condition with respect to $\boldsymbol{\beta}$ leads to

$$\boldsymbol{\beta}^* = \left[\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t + \rho\boldsymbol{\Sigma}\right]^{-1}\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\mu}, \tag{1.26}$$

where $\left[\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t + \rho\boldsymbol{\Sigma}\right]^{-1}$ is the inverse of an $m \times m$ matrix. We can infer from $\boldsymbol{\beta}^*$ that the objective of aggregating performance measures is to balance three effects: (i) the effort distortion characterized by $\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\mu}$, ($ii$) the measure-cost efficiency described by $\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t$; and ($iii$), the precision of the aggregated performance evaluation with the agent's risk tolerance, characterized by $\rho\boldsymbol{\Sigma}$.[14] The more risk averse the agent is, the more important becomes the latter effect for $\boldsymbol{\beta}$. Since these three effects are also determined by the agent's characteristics $\boldsymbol{\Psi}$ and ρ, we can conclude that the optimal aggregation of information is tied to individual agents. Roughly speaking, the principal tailors the aggregation of available performance measures to the specific characteristics of agents.

As mentioned earlier, the principal can influence the agent's effort allocation if she receives more than one performance measure. Note, however, that this is only feasible if at least two available measures do not contain the same information about the agent's relative effort allocation. Formally, for at least two performance measures $P_j(\boldsymbol{e}), P_k(\boldsymbol{e}) \in \mathbf{P}$ there exists no constant $\vartheta \neq 0$ satisfying $\boldsymbol{\omega}_j = \vartheta\boldsymbol{\omega}_k$, $j \neq k$. By combining these measures appropriately, the principal can—besides mitigating the uncertainty in the aggregated measure—improve the agent's effort allocation.

Proposition 1.6 *If there exist no constants $\vartheta_l \neq 0$ satisfying $\boldsymbol{\omega}_k = \vartheta_l\boldsymbol{\omega}_l$, $k \neq l$, $k, l \in \{1, ..., m\}$, for at least h performance measures with $n \leq h \leq m$, the principal can aggregate these measures such that the agent implements $\boldsymbol{e}^* = \lambda\boldsymbol{e}^{fb}$, $0 < \lambda \leq 1$. However, this is only optimal, if and only if,*

$$\rho\boldsymbol{\Sigma} = \hat{\lambda}\,\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t, \qquad \hat{\lambda} = \frac{1-\lambda}{\lambda}. \tag{1.27}$$

Proof See appendix.

The first condition in proposition 1.6 emphasizes that the principal needs access to an information system generating at least the same quantity of performance measures as number of tasks the agent has to perform.[15] Moreover, their sensitivity vectors are required to be linearly independent, i.e. performance measures differ in their information content with respect to the implemented effort allocation. If these two requirements are satisfied, the principal can combine these measures appropriately in order to motivate the agent to implement the first-best effort allocation. As the second condition in proposition 1.6 highlights, the aggregation of performance measures with the purpose of motivating the first-best effort allocation is only optimal if the covariance matrix $\boldsymbol{\Sigma}$ is a transformation of the measure-cost efficiency

[14] For a detailed analysis and discussion how performance measures are balanced in an aggregate, refer to Datar et al. [2001]. However, since they do not consider different task-specific abilities, their observations are slightly different in the sense that in their optimal aggregation the measure-cost efficiency does not play a role and therefore, effort distortion is only affected by the performance measure congruity.

[15] Note that this condition is sufficient, i.e. the principal can also induce a first-best effort allocation with less performance measures if e.g. one measure is perfectly congruent.

$\mathbf{\Xi}\mathbf{\Psi}^{-1}\mathbf{\Xi}^t$. In this case, aggregating performance measures to exclusively motivate the first-best effort allocation contemporaneously maximizes the precision of the aggregate, and consequently, minimizes the agent's risk premium. However, the most important observation is that (1.27) is also tied to the agent's characteristics $\mathbf{\Psi}$ and ρ. If the principal can employ a 'suitable' agent for a given information system, the optimal incentive contract may eventually motivate the first-best effort allocation.

Even though it might be optimal from the principal's perspective to provide the agent with incentives motivating the first-best effort allocation, it is not necessarily optimal that they contemporaneously induce a first-best effort intensity, as the next corollary to proposition 1.6 emphasizes.

Corollary 1.4 *Suppose there exist no constants $\vartheta_l \neq 0$ satisfying $\boldsymbol{\omega}_k = \vartheta_l \boldsymbol{\omega}_l$, $k \neq l$, $k, l \in \{1, ..., m\}$, for h performance measures with $n \leq h \leq m$. Then, it is optimal to induce $\mathbf{e}^{fb}$, if and only if, either $\rho = 0$ or $\mathbf{\Sigma} = [0]_{ij}$, $i, j = 1, ..., m$.*

Proof See appendix.

Consequently, the optimal linear incentive contract motivates the agent to implement a first-best effort allocation and intensity if two fundamental criteria are satisfied. First, the principal has access to at least the same quantity of appropriate performance measures as quantity of relevant tasks. These measures are required to provide different information about the implemented effort allocation. Second, either all performance measures are perfectly precise (i.e. noiseless) or the agent is risk-neutral. For single-task agencies, it is well known that the second criteria is sufficient to achieve first-best if the agent is not financially constrained. Multi-task agencies, however, impose additional requirements on the information system with respect to the characteristics and quantity of generated performance measures. In particular, the principal needs access to an information system which can be adjusted such that it reflects the agent's multidimensional contribution to gross payoff. Then, the principal can motivate the agent to conduct an efficient effort allocation by providing her congruent incentives.

1.8 Adverse Selection

The preceding analyses indicate that the properties of the agent's task-specific abilities play a crucial role for the design of incentive contracts and the value of employing particular agents. This offers the principal sufficient latitude to enhance her expected profit by applying adverse selection mechanisms aimed at choosing the 'most appropriate' agent for a given information system and set of tasks. The objective of this section is a brief illustration of adverse selection in multi-task agencies, when agents differ with respect to their task-specific abilities. The focus is thereby on the characteristics of the most beneficial type from the principal's perspective, rather than on the mechanism design itself.[16]

Suppose there exists a non-empty set of agents $\mathbf{A}$. Each agent $i \in \mathbf{A}$ is characterized by her individual task-specific abilities $\mathbf{\Psi}_i$ and risk tolerance $1/\rho_i$. For

[16] For adverse selection models refer e.g. to Salanié [1997] and Bolton and Dewatripont [2005], and the references therein. For adverse selection in a multi-task agency setting where agents' talents also affect their effort costs, see Moen and Rosen [2001].

simplicity, each agent knows her own type prior to signing the contract. The respective types are exogenous and do not change over time. The principal, however, can neither observe the agents' types nor does she receive any signals indicating the respective types, but she knows the distribution of available types in the economy. Accordingly, she can adjust the incentive contract such that only a desired type accepts, whereas less preferred types refuse. Precisely speaking, the principal sets the contract parameters α and $\boldsymbol{\beta}$ such that the participation constraint for a superior type $t_i(\boldsymbol{\Psi}_i, \rho_i)$, $i \in \mathbf{A}$, is binding, and violated for all less valuable types $j \in \mathbf{A}$, $j \neq i$. Suppose the principal wants to employ a type i and the corresponding incentive contract would also ensure the participation of another type k, with $i, k \in \mathbf{A}$. Then, two cases are possible. First, k's participation constraint is also binding, thereby implying k's employment as equally valuable as i's from the principal's perspective. Second, k's participation constraint is not binding so that she could extract an economic rent. If this is the case, we can infer that the employment of k is strictly superior and the principal is better off by tailoring the incentive contract to her characteristics.

Recall that the optimal linear incentive contract derived in section 1.7 implies that the participation constraint for a given type is binding. Thus, from an analytical perspective, it is sufficient to compare the expected profits induced by each available type in order to identify the 'most appropriate' one. A type $\hat{t}(\hat{\boldsymbol{\Psi}}, \hat{\rho})$ is therefore superior from the principal's perspective if her employment guarantees the highest of all feasible expected profits. Formally,

$$\hat{t}(\hat{\boldsymbol{\Psi}}, \hat{\rho}) \rightarrow \Pi(\hat{\boldsymbol{\Psi}}, \hat{\rho}) = \max \left\{ \Pi(\boldsymbol{\Psi}_i, \rho_i) \right\}_{i \in \mathbf{A}} . \tag{1.28}$$

Consequently, the principal tailors the incentive contract to her characteristics and provides the agent with $(\alpha(\hat{\boldsymbol{\Psi}}, \hat{\rho}), \boldsymbol{\beta}(\hat{\boldsymbol{\Psi}}, \hat{\rho}))$. Using the results from section 1.7, the problem can be formulated as

$$\hat{t}(\hat{\boldsymbol{\Psi}}, \hat{\rho}) \rightarrow \Pi(\hat{\boldsymbol{\Psi}}, \hat{\rho}) = \max \left\{ \frac{1}{2} \boldsymbol{\mu}^t \boldsymbol{\Psi}_i^{-1} \boldsymbol{\Xi}^t \left[\boldsymbol{\Xi} \boldsymbol{\Psi}_i^{-1} \boldsymbol{\Xi}^t + \rho_i \boldsymbol{\Sigma} \right]^{-1} \boldsymbol{\Xi} \boldsymbol{\Psi}_i^{-1} \boldsymbol{\mu} \right\}_{i \in \mathbf{A}} . \tag{1.29}$$

Identifying the superior type is not trivial since this condition depends on specific matrix products and the inverse of an $m \times m$ matrix. Nevertheless, the next proposition summarizes some inferences about the superior type satisfying (1.29).

Proposition 1.7 *Suppose there exists a non-empty set of agents* $\mathbf{A}$*, each of them characterized by* $t_i(\boldsymbol{\Psi}_i, \rho_i)$*,* $i \in \mathbf{A}$*. Then, the superior type* $\hat{t}(\hat{\boldsymbol{\Psi}}, \hat{\rho})$ *balances the following effects in the most efficient way:*

(i): The measure-cost efficiency effect characterized by $\boldsymbol{\Xi} \boldsymbol{\Psi}_i^{-1} \boldsymbol{\Xi}^t$*,*
(ii): The distortion effect characterized by $\boldsymbol{\mu}^t \boldsymbol{\Psi}_i^{-1} \boldsymbol{\Xi}^t$ *and its transpose,*
(iii): The risk effect characterized by $\rho_i \boldsymbol{\Sigma}$*.*

In principle, the value of particular agents depends—besides on their task-specific abilities and risk-aversion—on the subsequent job characteristics: (i) the number and properties of performance measures generated by an information system; and (ii), the relative contribution of all tasks to gross payoff. To exemplify the latter job characteristic, recall that $\boldsymbol{\mu}^t \boldsymbol{\Psi}_i^{-1} \boldsymbol{\Xi}^t$ emphasizes the effort distortion as a result of the information congruity relative to the agent's task-specific abilities $\boldsymbol{\Psi}_i$. Consider for instance two organizations k and l with identical information systems. They have different preferences for agents if there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu}_k = \lambda \boldsymbol{\mu}_l$. Otherwise, k's gross payoff function is (possibly) differently scaled than l's without affecting the induced effort distortion. In this case, the *distortion effect* is identical for both organizations, which leads to identical preferences for specific types.[17]

The relation between these emphasized effects provides two main implications for the selection of agents. First, organizations, or subunits, with different information systems may prefer different types, even if their gross payoff functions are identical. This observation follows directly from the *distortion effect*, *measure-cost efficiency effect* and *risk effect*. Second, organizations, or subunits, with different gross payoff functions may choose different types, even if they have access to identical information systems. This is implied by the *distortion effect*. Generally speaking, the individual value of available agents can only be assessed with respect to the corresponding job characteristics: (i) the relevant tasks and their contribution to firm's outcome; and (ii), the precision and congruity of the available information system. For illustrative purposes, consider for instance a manager and a worker sharing for simplicity the same risk tolerance. Due to prior learning experiences, the manager is assumed to exhibit relative higher abilities in performing administrative tasks than in conducting manufacturing related tasks. For the worker, however, the reversed relation is assumed. Now, who is superior from a firm's perspective? As previously emphasized, this cannot be assessed without considering the particular job characteristics. The manager is superior for jobs consisting primarily of administrative tasks, whereas it is efficient to employ the worker for manufacturing goods. As a result, both individuals are allocated to different jobs and obtain various incentive contracts tailored to their respective abilities and performance measurement. Now suppose it is desirable from the principal's perspective to employ two managers A and B characterized by the same risk tolerance. Assume that manager A exhibits a higher relative ability in performing administrative tasks than manager B, but the latter one can supervise her subordinates more effectively. The previous results indicate that the principal tailors the incentive contracts to their respective abilities. As a consequence, both managers receive different incentive contracts, even though they are in charge of performing identical tasks.

Proposition 1.8 *Let* $\mathbf{T} \subseteq \mathbf{A}$ *be the set of superior types. Then,* $\mathbf{T} \subseteq \mathbf{A}$ *can contain various types with* $t_k(\boldsymbol{\Psi}_k, \rho_k) \neq t_l(\boldsymbol{\Psi}_l, \rho_l)$, $k, l \in \mathbf{T} \subseteq \mathbf{A}$, $k \neq l$. *Nevertheless, it is possible that* $(\alpha^*(\boldsymbol{\Psi}_k, \rho_k), \boldsymbol{\beta}^*(\boldsymbol{\Psi}_k, \rho_k)) = (\alpha^*(\boldsymbol{\Psi}_l, \rho_l), \boldsymbol{\beta}^*(\boldsymbol{\Psi}_l, \rho_l))$, $k, l \in \mathbf{T} \subseteq \mathbf{A}$.

Proof See appendix.

This result highlights that the principal does not necessarily strictly prefer identical types of agents. That is, a type k can be equally valuable for the principal as type l, even though $t_k(\boldsymbol{\Psi}_k, \rho_k) \neq t_l(\boldsymbol{\Psi}_l, \rho_l)$, $k, l \in \mathbf{T} \subseteq \mathbf{A}$, $k \neq l$. Indifference

[17] Note that the same inference about two information systems characterized by $\boldsymbol{\Xi}_k = \lambda \boldsymbol{\Xi}_l$, $\lambda \neq 0$, cannot be made. This is due to their respective scale and its effect on the precision of the information system relative to the information content.

between different types of agents requires that some of them have a comparative disadvantage in one or two of the three dimensions emphasized by proposition 1.7, which is perfectly countervailed by a comparative advantage in the remaining dimension(s). To exemplify the second result emphasized by proposition 1.8, suppose the principal wants to employ several agents for jobs with identical characteristics. Then, the eventually employed agents are not necessarily identical, even though their jobs are similar. In any case, however, it is optimal to tailor their respective incentive contract to their individual characteristics. In general, one can expect to observe different contracts for various types of agents. Nonetheless, it is also possible that different agents receive identical incentive contracts.

1.9 Conclusion

Applying incongruent performance measures in incentive contracts motivates agents to implement an inefficient effort allocation across relevant tasks. This essay incorporates task-specific abilities in a multi-task agency framework and investigates their effects on the provision of incentives. As demonstrated, task-specific abilities determine the efficiency of the agent's effort allocation and play an important role for the contractual design.

When the principal applies incongruent and noisy performance measures in incentive contracts, the agent's effort choice deviates from first-best with respect to two dimensions. First, as well known, the optimal incentive contract induces a suboptimal effort intensity due to the agent's desire for insurance. Second, the agent chooses an inefficient effort allocation if the performance measure does not reflect her contribution to gross payoff. The extent of the latter inefficiency, however, depends on the agent's task-specific abilities relative to the performance measure congruity. As a result, incentive contracts are tailored to the agent's abilities and, particularly, depend on three factors: (*i*) the inefficiency of effort distortion as a result of applying incongruent performance measures in incentive contracts, relative to the agent's task-specific abilities (*distortion effect*), (*ii*) the agent's effort costs associated with the motivated effort allocation (*measure-cost efficiency*); and (*iii*), the precision of the information system with the agent's risk-aversion (*risk effect*).

This essay further proposes a ranking criteria for performance measures in multi-task agencies. One important observation is that the signal/noise ratio, commonly used to assess performance measures in single-task agencies, is not a sufficient ranking criteria in multi-task agencies. The relative value of performance measures depends—besides on their precision—on their congruity relative to the agent's task-specific abilities, thereby implying that their ranking is tied to the agent's characteristics. The same is true for the optimal aggregation of multiple performance measures. As further illustrated, the principal can motivate the agent to implement a first-best effort allocation if she has access to a sufficient quantity of appropriate performance measures. However, this is only optimal if the efficient aggregation maximizes the precision of the information system while motivating the (agent-specific) first-best effort allocation.

The characteristics of agents, particularly their task-specific human capital, do not only affect their performance evaluation and incentive contracts, they also determine the benefit of their employment from the principal's perspective. It is consequently in the principal's interest to apply adverse selection mechanisms to guarantee the employment of the most valuable agent. As shown, the best available type of agent balances three effects most efficiently: (*i*) the *distortion effect*, (*ii*) the *measure-cost efficiency effect*; and (*iii*), the *risk effect*. Due to the characteristics of these effects, the value of individual agents is linked to the respective set of tasks the agent is in charge of, and attributes of the information system. Different agents, however, may be equally valuable, but may, nonetheless, receive different incentive contracts. Generally speaking, task-specific abilities and the properties of information systems can explain why different agents are allocated to various jobs; or why they receive different incentive contracts, even if their jobs are identical.

This essay is part of a larger research agenda. Previous multi-task literature focused primarily on performance measure congruity and its effect on incentive contracts. As this essay illustrates, we can shed more light on the nature of incentive contracts in multi-task agency relations, when we keep in mind that agents may differ in their skills and abilities to perform particular tasks. I believe it is substantial to further explore the effects of task-specific human capital on incentive contracts and the optimal selection of agents. In particular, if task-specific abilities change over time due to work experience, and the principal cannot precisely observe this mutation, she will update her beliefs about the individual abilities in accordance to the agent's prior performances. Such framework could contribute to our understanding of the dynamics of incentive contracts. However, I leave these fascinating issues for future research.

1.10 Appendix

Proof of Proposition 1.1.
Recall that $\boldsymbol{e}^* = \boldsymbol{\Gamma}\beta$. Consider first a change in β. Without loss of generality, choose two arbitrary activities e_j and e_k, $j, k \in \{1, ..., n\}$, $j \neq k$. A variation of β influences the effort intensity without the effort allocation, if $\lambda = e_k/e_j$ is independent of β. Substituting $e_i = \Gamma_i\beta$, $i = j, k$, gives $\lambda = \Gamma_k/\Gamma_j$. Accordingly, the relative effort allocation cannot be adjusted by β. In contrast, β affects the effort intensity if there exists no constant $\lambda \neq 0, 1$ satisfying $\boldsymbol{e}(\beta') = \lambda\boldsymbol{e}(\beta'')$ for two arbitrary incentive parameters $\beta' \neq \beta''$. Consequently, $\boldsymbol{\Gamma}\beta' = \lambda\boldsymbol{\Gamma}\beta''$, which is equivalent to $\lambda\boldsymbol{I} = (\beta'/\beta'')\,\boldsymbol{I}$, where $\boldsymbol{I}$ is an $n \times n$ identity matrix. Hence, the effort intensity can be adjusted by β.
Next, consider again two arbitrary activities e_j and e_k, $j, k \in \{1, ..., n\}$, $j \neq k$, the agent implements for task j and k given ω_i and ψ_i, $i = j, k$. Additionally, consider two activities $\bar{e}_j$ and $\bar{e}_k$ the agent implements for task j and k given $\bar{\omega}_i$ and $\bar{\psi}_i$, $i = j, k$. Then, the relative effort allocation changes if $e_j/e_k \neq \bar{e}_j/\bar{e}_k$, which is equivalent to

$$\frac{\omega_j\psi_k}{\omega_k\psi_j} \neq \frac{\bar{\omega}_j\bar{\psi}_k}{\bar{\omega}_k\bar{\psi}_j}. \tag{1.30}$$

Observe that the relative effort allocation changes if $\psi_i \neq \bar{\psi}_i$ and/or $\omega_i \neq \bar{\omega}_i$, $i = j, k$, except for the case where (1.30) is an equality even though $\psi_i \neq \bar{\psi}_i$ and/or $\omega_i \neq \bar{\omega}_i$, $i = j, k$. Thus, the effort allocation depends on $\boldsymbol{\Gamma} = \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}$.

Q.E.D.

Proof of Proposition 1.2.
Effort distortion refers to the relation of $\boldsymbol{e}^*$ to $\boldsymbol{\mu}$ and can be therefore measured by the vector product $\boldsymbol{\mu}^t\boldsymbol{e}^*$. Since $\boldsymbol{e}^* = \boldsymbol{\Gamma}\beta$,

$$\boldsymbol{\mu}^t\boldsymbol{e} = \beta\sum_{i=1}^{n}\mu_i\Gamma_i = \beta\|\boldsymbol{\mu}\|\|\boldsymbol{\Gamma}\|\cos\theta. \tag{1.31}$$

First note that $\|\boldsymbol{\mu}\|$ does not affect the relative importance of tasks for $V(\boldsymbol{e})$. Furthermore, $\beta\|\boldsymbol{\Gamma}\|$ determines the lengths of vector $\boldsymbol{e}^*$, but not its direction in the n-dimensional space. The length is arbitrary in the sense that it can be adjusted by β. Consequently, $\Upsilon^D(\theta) = \cos\theta \in [0, 1]$ measures the induced effort distortion under second-best.

Q.E.D.

Proof of Proposition 1.3.
To measure effort distortion, we can use the vector product $\boldsymbol{\mu}^t\boldsymbol{e}^*$. If $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, then $\boldsymbol{e}^* = \beta\boldsymbol{\omega}/\hat{\psi}$. This leads to

$$\boldsymbol{\mu}^t\boldsymbol{e} = \frac{\beta}{\hat{\psi}}\sum_{i=1}^{n}\mu_i\omega_i = \frac{\beta}{\hat{\psi}}\|\boldsymbol{\mu}\|\|\boldsymbol{\omega}\|\cos\varphi. \tag{1.32}$$

Again, $\|\boldsymbol{\mu}\|$ does not affect the relative importance of tasks for $V(\boldsymbol{e})$, and $\beta\|\boldsymbol{\omega}\|$ determines the lengths of vector $\boldsymbol{e}^*$ but not its direction in the n-dimensional space. Thus, $\bar{\Upsilon}^D(\varphi) = \cos\varphi \in [0,1]$ measures distortion under second-best if $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. Consequently, $\bar{\Upsilon}^D(\varphi) = \Upsilon^C(\varphi)$.

Q.E.D.

Proof of Corollary 1.3.
If $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, then $\boldsymbol{\Gamma}_i = \boldsymbol{\omega}_i/\hat{\psi}$ and $\|\boldsymbol{\Gamma}_i\| = \|\boldsymbol{\omega}_i\|/\hat{\psi}$, $i = \{j, k\}$. Consequently, $\Upsilon^{M/C}(\xi = 0) = 1$ and $\bar{\Upsilon}^D(\varphi_i) = \Upsilon^C(\varphi_i)$, see proposition 1.3. By substituting $\Lambda_i = \|\boldsymbol{\omega}_i\|^2/\sigma_i^2$, $i = \{j, k\}$, the ranking criteria of proposition 1.4 can be reformulated to the one stated in the corollary.

Q.E.D.

Proof of Proposition 1.5.
Observe first that the expected profit on the basis of $P_i(\boldsymbol{e})$ can be written as

$$\Pi^* = \frac{(\boldsymbol{\mu}^t\boldsymbol{\Gamma}_i)^2}{2(\boldsymbol{\omega}_i^t\boldsymbol{\Gamma}_i + \rho\sigma_i^2)}. \tag{1.33}$$

Recall that $\boldsymbol{\Gamma}_i = \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_i$. Consequently, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $\forall j \neq k$, if and only if,

$$\frac{\left(\boldsymbol{\mu}^t\boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_k\right)^2}{2(\boldsymbol{\omega}_k^t\boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_k + \rho\sigma_k^2)} > \frac{\left(\boldsymbol{\mu}^t\boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_j\right)^2}{2(\boldsymbol{\omega}_j^t\boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_j + \rho\sigma_j^2)}. \tag{1.34}$$

If $\boldsymbol{\omega}_k = \lambda\boldsymbol{\omega}_j$, we can re-scale $P_j(\boldsymbol{e})$ such that it is characterized by the same sensitivity in $\boldsymbol{e}$ as $P_k(\boldsymbol{e})$. Accordingly,

$$\bar{P}_j(\boldsymbol{e}) = \boldsymbol{\omega}_j^t\boldsymbol{e} + \frac{\varepsilon_j}{\lambda}, \tag{1.35}$$

where var $\left[\bar{P}_j(\boldsymbol{e})\right] = \sigma_j^2\lambda^{-2}$. Let $\boldsymbol{\omega} \equiv \boldsymbol{\omega}_i$, $i = j, k$. This leads to

$$\frac{\left(\boldsymbol{\mu}^t\boldsymbol{\Psi}^{-1}\boldsymbol{\omega}\right)^2}{2(\boldsymbol{\omega}^t\boldsymbol{\Psi}^{-1}\boldsymbol{\omega} + \rho\sigma_k^2)} > \frac{\left(\boldsymbol{\mu}^t\boldsymbol{\Psi}^{-1}\boldsymbol{\omega}\right)^2}{2(\boldsymbol{\omega}^t\boldsymbol{\Psi}^{-1}\boldsymbol{\omega} + \rho\sigma_j^2\lambda^{-2})}, \tag{1.36}$$

which can be re-arranged to

$$\frac{1}{\sigma_k^2} > \frac{\lambda^2}{\sigma_j^2}. \tag{1.37}$$

Recall that after re-scaling, $\boldsymbol{\omega}_k = \boldsymbol{\omega}_j$. Thus, (1.37) can be written as

$$\frac{\|\boldsymbol{\omega}_k\|^2}{\sigma_k^2} > \frac{\lambda^2\|\boldsymbol{\omega}_j\|^2}{\sigma_j^2}, \tag{1.38}$$

which is identical to $\Lambda_k > \Lambda_j$.

Q.E.D.

Proof of Proposition 1.6.
The agent implements the first-best effort allocation, if $\boldsymbol{e}^* = \lambda \boldsymbol{e}^{fb}$. Note, however, that $0 < \lambda \leq 1$ since it cannot be optimal to induce a higher effort intensity under second-best than under first-best. Therefore, $\boldsymbol{\beta}$ needs to solve $\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t\boldsymbol{\beta} = \lambda\boldsymbol{\Psi}^{-1}\boldsymbol{\mu}$, which is equivalent to $\boldsymbol{\Xi}^t\boldsymbol{\beta} = \lambda\boldsymbol{\mu}$. If $\operatorname{rank}\boldsymbol{\Xi}^t \geq n$, there exists at least one solution of this equation system. In particular, h columns in $\boldsymbol{\Xi}^t$, $n \leq h \leq m$, must be linearly independent. Consequently, $\operatorname{rank}\boldsymbol{\Xi}^t \geq n$, if there exist no constants $\vartheta_l \neq 0$ satisfying $\boldsymbol{\omega}_k = \vartheta_l\boldsymbol{\omega}_l$, $k, l \in \{1, ..., m\}$, $k \neq l$, for h performance measures with $n \leq h \leq m$.
Inducing a first-best effort allocation is only optimal if $\boldsymbol{e}(\boldsymbol{\beta}^*) = \lambda\boldsymbol{e}^{fb}$. This particulary requires that $\boldsymbol{\Xi}^t\boldsymbol{\beta}^* = \lambda\boldsymbol{\mu}$, or equivalently, $\boldsymbol{\beta}^* = \lambda\left[\boldsymbol{\Xi}^t\right]^{-1}\boldsymbol{\mu}$. Substituting $\boldsymbol{\beta}^*$ gives

$$\begin{aligned}
\left[\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t + \rho\boldsymbol{\Sigma}\right]^{-1}\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\mu} &= \lambda\left[\boldsymbol{\Xi}^t\right]^{-1}\boldsymbol{\mu} && (1.39)\\
\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1} &= \lambda\left[\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t + \rho\boldsymbol{\Sigma}\right]\left[\boldsymbol{\Xi}^t\right]^{-1} && (1.40)\\
\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1} &= \lambda\boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t\left[\boldsymbol{\Xi}^t\right]^{-1} + \lambda\rho\boldsymbol{\Sigma}\left[\boldsymbol{\Xi}^t\right]^{-1} && (1.41)\\
(1-\lambda)\ \boldsymbol{\Xi}\boldsymbol{\Psi}^{-1} &= \lambda\rho\boldsymbol{\Sigma}\left[\boldsymbol{\Xi}^t\right]^{-1}, && (1.42)
\end{aligned}$$

which is equivalent to

$$\rho\boldsymbol{\Sigma} = \frac{1-\lambda}{\lambda}\ \boldsymbol{\Xi}\boldsymbol{\Psi}^{-1}\boldsymbol{\Xi}^t. \tag{1.43}$$

Q.E.D.

Proof of Proposition 1.8.
Suppose a type $\hat{t}(\hat{\boldsymbol{\Psi}}, \hat{\rho})$ satisfies (1.29). Assume further that there exists another type $t_i(\boldsymbol{\Psi}_i, \rho_i)$ satisfying $\Pi(\hat{\boldsymbol{\Psi}}, \hat{\rho}) = \Pi(\boldsymbol{\Psi}_i, \rho_i)$. This implies

$$\boldsymbol{\mu}^t\hat{\boldsymbol{\Psi}}^{-1}\boldsymbol{\Xi}^t\left[\boldsymbol{\Xi}\hat{\boldsymbol{\Psi}}^{-1}\boldsymbol{\Xi}^t + \hat{\rho}\boldsymbol{\Sigma}\right]^{-1}\boldsymbol{\Xi}\hat{\boldsymbol{\Psi}}^{-1}\boldsymbol{\mu} = \boldsymbol{\mu}^t\boldsymbol{\Psi}_i^{-1}\boldsymbol{\Xi}^t\left[\boldsymbol{\Xi}\boldsymbol{\Psi}_i^{-1}\boldsymbol{\Xi}^t + \rho_i\boldsymbol{\Sigma}\right]^{-1}\boldsymbol{\Xi}\boldsymbol{\Psi}_i^{-1}\boldsymbol{\mu}. \tag{1.44}$$

We know that $\hat{t}(\hat{\boldsymbol{\Psi}}, \hat{\rho})$ is exogenous but we can treat agent i's characteristics $t_i(\boldsymbol{\Psi}_i, \rho_i)$ as endogenous in order to show that there can be several types satisfying (1.44). Accordingly, we have an equation with $n+1$ independent variables. Thus, depending on the parameter values, there can be several types satisfying (1.44).
Finally observe that different types generally lead to different incentive contracts. However, to proof that $(\alpha^*(\boldsymbol{\Psi}_k, \rho_k), \boldsymbol{\beta}^*(\boldsymbol{\Psi}_k, \rho_k)) \neq (\alpha^*(\boldsymbol{\Psi}_l, \rho_l), \boldsymbol{\beta}^*(\boldsymbol{\Psi}_l, \rho_l))$ is not always true, even though $t_k(\boldsymbol{\Psi}_k, \rho_k) \neq t_l(\boldsymbol{\Psi}_l, \rho_l)$, $k, l \in \mathbf{T} \subseteq \mathbf{A}$, $k \neq l$, I subsequently provide a counter example. Suppose the principal receives one performance measure $P(\boldsymbol{e})$. Assume that two types $t_k(\boldsymbol{\Psi}_k, \rho_k) \neq t_l(\boldsymbol{\Psi}_l, \rho_l)$, $k, l \in \mathbf{T} \subseteq \mathbf{A}$, $k \neq l$, satisfy (1.29), and they do not exhibit cost substitutes or complements, thereby implying that $\boldsymbol{\Psi}_l$ and $\boldsymbol{\Psi}_k$ are diagonal matrices. The optimal incentive parameters β_k^* and β_l^* for agent k and agent l, respectively, are

$$\beta_k^* = \frac{\displaystyle\sum_{i=1}^{n}\frac{\mu_i\omega_i}{\psi_{ki}}}{\displaystyle\sum_{i=1}^{n}\frac{\omega_i\omega_i}{\psi_{ki}} + \rho_k\sigma^2} \qquad \beta_l^* = \frac{\displaystyle\sum_{i=1}^{n}\frac{\mu_i\omega_i}{\psi_{li}}}{\displaystyle\sum_{i=1}^{n}\frac{\omega_i\omega_i}{\psi_{li}} + \rho_l\sigma^2},$$

where ψ_{ji} denotes agent j's task-specific ability with respect to task i, $j = k, l$. Observe that $\beta_k^* = \beta_l^*$, if e.g. $n = 2$, $\mu_1 = \mu_2$, $\omega_1 = \omega_2$ and both agents are further characterized by $\rho_k = \rho_l$ and $\psi_{k1} = \psi_{l2}$ and $\psi_{k2} = \psi_{l1}$, $k \neq l$. Since $\beta_k^* = \beta_l^*$ and $\rho_k = \rho_l$, the risk premium is identical for both agents. Although each agent implements a different effort allocation with $\boldsymbol{e}_k^* = (\omega_1\beta_k^*/\psi_{k1}, \omega_2\beta_k^*/\psi_{k2})^t$ and $\boldsymbol{e}_l^* = (\omega_1\beta_l^*/\psi_{l1}, \omega_2\beta_l^*/\psi_{l2})^t$, observe that $C(\boldsymbol{e}_k^*) = C(\boldsymbol{e}_l^*)$ since $e_{k1}^* = e_{l2}^*$ and $e_{k2}^* = e_{l1}^*$. As a result, $\alpha^*(\boldsymbol{\Psi}_k, \rho_k) = \alpha^*(\boldsymbol{\Psi}_l, \rho_l)$. If this is possible for a single performance measure and two-dimensional effort, it can be also the case for multiple measures and $n > 2$. Hence, even though two types $t_k(\boldsymbol{\Psi}_k, \rho_k) \neq t_l(\boldsymbol{\Psi}_l, \rho_l)$ satisfy condition (1.29), it can be true that $(\alpha^*(\boldsymbol{\Psi}_k, \rho_k), \boldsymbol{\beta}^*(\boldsymbol{\Psi}_k, \rho_k)) = (\alpha^*(\boldsymbol{\Psi}_l, \rho_l), \boldsymbol{\beta}^*(\boldsymbol{\Psi}_l, \rho_l))$, $k, l \in \mathbf{T} \subseteq \mathbf{A}$.

Q.E.D.

Essay 2

Costly Performance Measurement in Multi-Task Agencies

2.1 Introduction

Many employees are charged with performing multiple tasks which may differently contribute to firms' objectives. Employees can therefore not only decide on their effort intensity, but also on how to allocate their effort across all relevant tasks. If effort is non-contractible, firms face a two-dimensional incentive problem: they need not only to induce a sufficient effort intensity, but also to motivate an efficient effort allocation across tasks. The latter objective could be achieved if firms are able to identify each employee's individual contribution and apply this information in incentive contracts. However, company structures are often too complex for accrediting the respective contribution to each employee. As a consequence, firms are compelled to employ other potential incentive mechanisms.

One alternative is the application of objective performance measures in incentive contracts, as extensively analyzed in agency literature.[1] However, the application of performance measures in incentive contracts can motivate employees to focus on activities which suitably enhance their performance evaluation, but have possibly little or even negative effects on firm value. The behavioral literature provides illustrative examples of such dysfunctional behavior. For instance, Prendergast [1999] reported that AT&T rewarded their software engineers for the quantity of lines they wrote for their programs. It was soon discovered that the programs consisted of more lines than necessary.

Even if all relevant activities contribute to firm value, the relative effort allocation across those tasks can be inefficient. This occurs, when firms apply performance measures to provide their employees' with incentives, which do not perfectly reflect their individual contribution to firm value. The provision of incentives based on such *incongruent* performance measures leads therefore to a suboptimal effort allocation across tasks [Feltham and Xie, 1994]. For example, faculties at universities are primarily responsible for two tasks: teaching and conducting research. Since teaching is harder to quantify than the output from research, promotion decisions are generally

[1]For a review of agency literature refer e.g. to Prendergast [1999], Lambert [2001], Gibbons [2005], and Christensen and Feltham [2005].

made on the basis of research accomplishments. This in turn motivates particularly younger faculties to concentrate on research at the expense of teaching.[2] Nevertheless, schools can modify their incentive schemes in order to improve the quality of teaching. Brickley and Zimmerman [2001] considered the incentive scheme adapted by the William E. Simon Graduate School of Business Administration, University of Rochester. After adjusting the performance evaluation and the reward system during the early 90's, teaching quality improved significantly, but at the same time, research output declined.

If organizations do not have access to performance evaluations that are suitable to induce an efficient effort allocation, they need to apply alternative mechanisms that act to mitigate effort distortion. Recent multi-task agency literature analyzes and discusses some alternatives to improve agents' effort allocations. For instance, Feltham and Xie [1994], Banker and Thevaranjan [2000], and Datar et al. [2001] analyze, how multiple performance measures can be aggregated to mitigate effort distortion besides alleviating incentive risk.[3] As demonstrated in chapter 1, the principal can even motivate the agent to implement non-distorted effort if she has access to a sufficient quantity of appropriate performance measures. A second stream of the multi-task agency literature focuses on the optimal design of jobs as a device to restrict effort distortion by allowing independent tasks to be split among multiple agents, see e.g. Holmström and Milgrom [1991], Schöttner [2005], Corts [2005], and Hughes et al. [2005]. Finally, Holmström [1999] and Baker et al. [2002] emphasize that transferring asset ownership to the agent can mitigate her effort distortion because asset ownership internalizes the consequences of her effort allocation. Nevertheless, such an approach compromises the objective of efficient risk-sharing if the agent is risk-averse and the principal is risk-neutral.

The ability to improve employees' performance evaluations in order to mitigate their effort distortion can be crucial for the efficiency of firms. To understand the provision of incentives in multi-task agency relations, it is essential to investigate whether and how organizations respond to the lack of perfectly congruent performance measures, and consequently, to the imposed inefficiencies due to effort distortion. Surprisingly, previous multi-task literature remains virtually silent about costly performance measurement with the objective of improving the agent's effort allocation. This essay thus focuses on costly performance measurement in order to glean new insights into the improvement of information systems aimed at mitigating effort distortion.

Specifically, I analyze a multi-task agency framework with risk-neutral parties, where the agent faces a liability limit constraint. Similar to Baker [2002], I adopt geometric representations for performance measure congruity, which eventually form the foundation for the considered measurement technology. The main emphasis lies in understanding the principal's preference for either investing centrally in assets which provides a viable means of measuring the agent's performance, or delegating the information acquisition to a supervisor. However, employing a supervisor induces

[2]See Kerr [1975] for a discussion of this and further examples. Additional illustrative examples are summarized by Gibbons [1998] and Baker [2000].

[3]For an empirical investigation on how performance measures are combined, refer to Gibbs et al. [2004].

a second moral hazard problem since the principal cannot contract over her effort to acquire information.

The analysis in this essay indicates that recruiting a supervisor for measuring the agent's performance can only be beneficial if she provides a sufficient relative measurement efficiency, which countervails the additionally imposed agency costs. More precisely, the principal's decision on whether to delegate the information acquisition to the supervisor is contingent on three factors: (*i*) the precision of the supervisor's evaluation system, (*ii*) the supervisor's comparative cost advantage in obtaining the required information; and (*iii*), the congruence of the costless available information system about the agent's effort. If the supervisor's performance evaluation is sufficiently precise, an adequately incongruent costless information system generally favors delegation. The rationale for this observation is that a less congruent costless information system imposes lower requirements on the supervisor's relative measurement efficiency, which is more likely to be satisfied by a potential supervisor. In contrast, a more congruent information system would impose higher requirements on the supervisor's comparative cost advantage, which is less likely to be achieved by a potential supervisor. In this case, a centralized investment is presumably to be observable. If the supervisor's performance evaluation is sufficiently imprecise, the contrary implications apply.

This essay is closely related to the multi-task agency literature analyzing the efficiency of induced effort allocations, especially to Holmström and Milgrom [1991], Feltham and Xie [1994], Datar et al. [2001], and Baker [2002]. However, it deviates in two main directions. First, it utilizes a framework with a risk-neutral and financially constrained agent, i.e. payments from the agent to the principal are not feasible. This allows to abstract from risk considerations in order to focus exclusively on the induced effort allocation. Second, the analyzed incentive contract is one of the bonus type similar to the ones applied by Park [1995], Kim [1997], Pitchford [1998] and Demougin and Fluet [2001] for single-task agency relations.

This essay contributes to the previous multi-task agency literature in two important ways. First, it extends their work by investigating a costly mechanism aimed at improving the agent's performance evaluation and hence, the efficiency of her motivated effort allocation. Second, this essay provides preliminary insights into the relationship between the properties of available information systems and the optimal design of organizations aimed at efficiently improving these information systems.

The essay proceeds as follows. I introduce the basic model in section 2.2 and provide the first-best solution in section 2.3. In section 2.4, I derive and discuss the second-best contract and elaborate on a ranking criteria for information systems in multi-task agencies with risk-neutral parties. Subsequently, I analyze in section 2.5 the principal's investment decision for generating additional measures about the agent's performance. Section 2.6 focuses on the contractual arrangement, when the principal employs a supervisor who is charged with the acquisition of the required performance measures. Both considered alternatives for improving the underlying information system are compared in section 2.7. Section 2.8 summarizes the main results and concludes.

2.2 The Model

Consider a single-period agency relationship between a principal and an agent. Both parties are risk-neutral and the agent faces a liability limit constraint, i.e. payment from the agent to the principal are not feasible. The agent is assigned to perform $n > 2$ tasks which cannot be split among different agents. Therefore, she needs to implement a vector of effort $\boldsymbol{e} = (e_1, ..., e_n)^t$, $\boldsymbol{e} \in \mathbf{E} \subseteq \mathbb{R}^{n+}$, where e_i is the agent's effort allocated to task i.[4] Effort is non-verifiable and all activities $e_i \in \mathbf{E}$ are measured in the same unit. The agent's disutility of effort $C(\boldsymbol{e})$ is quadratic and separable in the different activities:

$$C(\boldsymbol{e}) = \sum_{i=1}^{n} \frac{1}{2} e_i^2 = \frac{1}{2} \boldsymbol{e}^t \boldsymbol{e}. \tag{2.1}$$

By implementing effort $\boldsymbol{e}$, the agent can affect the firm value V, which can be either high or low. Formally, let $V \in \{0, 1\}$, whereas the probability for realizing the high firm value conditional on $\boldsymbol{e}$ is

$$\text{Prob}\{V = 1|\boldsymbol{e}\} = \min\{\boldsymbol{\mu}^t \boldsymbol{e}, 1\}. \tag{2.2}$$

Vector $\boldsymbol{\mu} = (\mu_1, ..., \mu_n)^t$, $\boldsymbol{\mu} \in \mathbb{R}^{n+}$, represents the sensitivity of the expected firm value in the agent's effort. Accordingly, the agent can not only influence the expected firm value by her effort intensity, but also by her relative effort allocation across relevant tasks. To ensure an interior solution for the first-best effort vector $\boldsymbol{e}^{fb}$, I assume that $\boldsymbol{\mu}$ is characterized such that $\text{Prob}\{V = 1|\boldsymbol{e}^{fb}\} = \boldsymbol{\mu}^t \boldsymbol{e}^{fb} < 1$.[5]

The realized firm value is non-contractible, and therefore, cannot be used to provide the agent with explicit incentives. Nevertheless, the principal receives a binary and verifiable signal $\bar{S} \in \{0, 1\}$, where $\bar{S} = 1$ is the favorable signal in the sense of Milgrom [1981]. The probability of realizing the favorable signal is conditional on the agent's effort and takes the form

$$\text{Prob}\{\bar{S} = 1|\boldsymbol{e}\} = \min\{\bar{\boldsymbol{\omega}}^t \boldsymbol{e}, 1\}, \tag{2.3}$$

where $\bar{\boldsymbol{\omega}} = (\bar{\omega}_1, ..., \bar{\omega}_n)^t$, $\bar{\boldsymbol{\omega}} \in \mathbb{R}^{n+}$, represents the sensitivity of the expected signal in the agent's effort. This binary statistic potentially represents several costless performance measures which are summarized in the most efficient way. Henceforth, I refer to $\bar{S}$ as the costless information system. To ensure interior solutions, I shall assume that $\bar{\boldsymbol{\omega}}$ is characterized such that $\text{Prob}\{\bar{S} = 1|\boldsymbol{e}^*\} = \bar{\boldsymbol{\omega}}^t \boldsymbol{e}^* < 1$ for the second-best effort vector $\boldsymbol{e}^*$.[6]

Since the realized value of $\bar{S}$ is verifiable, the principal can exploit this information in a bonus contract to provide the agent with incentives to implement effort. Particulary, the agent obtains a bonus β^A in addition to a fixed transfer α^A if the favorable signal $\bar{S} = 1$ is realized. Accordingly, the agent's binary wage w^A takes the form

$$w^A = \begin{cases} \alpha^A + \beta^A, & \text{if } \bar{S} = 1, \\ \alpha^A, & \text{if } \bar{S} = 0. \end{cases} \tag{2.4}$$

[4] All vectors are column vectors where 't' denotes the transpose.

[5] By using the subsequently derived first-best solution, one can show that this requires $\boldsymbol{\mu}^t \boldsymbol{\mu} < 1$.

[6] One can show by using the subsequently derived second-best solution that this requires $\boldsymbol{\mu}^t \bar{\boldsymbol{\omega}}/2 < 1$.

As a result of the agent's liability limit, all transfers have to be non-negative for any realization of $\bar{S}$. If the agent accepts this bonus contract on the basis of $\bar{S}$, it provides her with the expected utility

$$U^A(\boldsymbol{e}) = \alpha^A + \beta^A \bar{\boldsymbol{\omega}}^t \boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t \boldsymbol{e}. \tag{2.5}$$

For parsimony, the agent's reservation utility is normalized to zero.

2.3 The First-Best Contract

Before investigating the second-best contract, it is useful to derive the first-best solution of this problem as a benchmark for the subsequent analysis. Suppose the principal can directly contract over $\boldsymbol{e}$. Then, she appoints the effort vector $\boldsymbol{e}^{fb}$ which maximizes the difference between the expected firm value $\mathrm{E}[V|\boldsymbol{e}]$ and wage payment $w^A = C(\boldsymbol{e})$. Formally, the first-best effort vector $\boldsymbol{e}^{fb}$ solves

$$\max_{\boldsymbol{e}} \Pi(\boldsymbol{e}) \equiv \boldsymbol{\mu}^t \boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t \boldsymbol{e}. \tag{2.6}$$

The first-order condition leads to $\boldsymbol{e}^{fb} = \boldsymbol{\mu}$. The principal assigns each activity e_i in accordance to its marginal effect μ_i on the expected firm value. To exemplify the relative effort allocation across tasks, consider the relation between two arbitrary activities e_i^{fb} and e_j^{fb}, $i \neq j$:

$$\frac{e_i^{fb}}{e_j^{fb}} = \frac{\mu_i}{\mu_j} \qquad i, j = 1, ..., n \;, \;\; i \neq j. \tag{2.7}$$

Suppose $\mu_i > \mu_j$. In this case, it is optimal to assign more effort on task i relative to task j since the former task contributes more to the expected firm value than the latter.[7] This implies that an implemented effort allocation is efficient if it reflects the relative marginal contribution of each task to the expected firm value. The conducted effort allocation is thus deemed to be *non-distorted*. In contrast, if an implemented effort allocation deviates from the one assigned under first-best, it is *distorted*. Formally, implemented effort is distorted if there exists no constant $\lambda > 0$ satisfying $\boldsymbol{e} = \lambda \boldsymbol{e}^{fb}$.

Finally, substituting $\boldsymbol{e}^{fb}$ in the principal's objective function leads to

$$\Pi^{fb} = \frac{1}{2}\boldsymbol{\mu}^t \boldsymbol{\mu}. \tag{2.8}$$

The principal's expected first-best profit is therefore only characterized by the vector product $\boldsymbol{\mu}^t \boldsymbol{\mu}$, where $\boldsymbol{\mu}$ represents the sensitivity of the expected firm value in the agent's effort.

[7] Note that this observation applies because marginal effort costs are assumed to be identical across relevant tasks. If they differ, the task-specific marginal effort costs relative to the respective marginal contribution to firm value determine the first-best effort allocation, see chapter 1.

2.4 The Second-Best Contract

If the principal cannot directly contract over $\boldsymbol{e}$, she faces an incentive problem. Since the firm value V is non-contractible, she is compelled to use the information system $\bar{S}$ to provide the agent with incentives. However, the application of $\bar{S}$ in an incentive contract can motivate the agent to implement an inefficient effort allocation across tasks if $\mathrm{E}[\bar{S}]$ does not perfectly reflect her contribution to the expected firm value $\mathrm{E}[V]$. In this case, the information system $\bar{S}$ is deemed to be *incongruent* with the expected firm value, and its application in the agent's incentive contract imposes *incongruent incentives*.

In a second-best environment, the principal's problem is to find a bonus contract $(\alpha^{A*}, \beta^{A*})$ aimed at maximizing her expected profit $\Pi \equiv \mathrm{E}[V - w^A|\boldsymbol{e}]$ while ensuring the agent's participation. Formally, the optimal bonus contract solves

$$\max_{\alpha^A, \beta^A, \boldsymbol{e}} \Pi \equiv \boldsymbol{\mu}^t \boldsymbol{e} - \alpha^A - \beta^A \bar{\boldsymbol{\omega}}^t \boldsymbol{e} \tag{2.9}$$

s.t.

$$\alpha^A + \beta^A \bar{\boldsymbol{\omega}}^t \boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t \boldsymbol{e} \geq 0 \tag{2.10}$$

$$\boldsymbol{e} = \arg\max_{\tilde{\boldsymbol{e}}} \alpha^A + \beta^A \bar{\boldsymbol{\omega}}^t \tilde{\boldsymbol{e}} - \frac{1}{2}\tilde{\boldsymbol{e}}^t \tilde{\boldsymbol{e}} \tag{2.11}$$

$$\alpha^A + \beta^A \geq 0 \tag{2.12}$$

$$\alpha^A \geq 0. \tag{2.13}$$

Condition (2.10) is the agent's participation constraint and ensures that it is in her interest to enter into this relationship. Moreover, (2.11) is the agent's incentive condition. Finally, (2.12) and (2.13) guarantee that the optimal bonus contract is compatible with the agent's liability limit.

We can directly infer from the agent's incentive constraint that she implements

$$\boldsymbol{e}^* = \beta^A \bar{\boldsymbol{\omega}}. \tag{2.14}$$

The second-best effort vector consists of two components: the scalar β^A and the vector $\boldsymbol{\mu}$. The bonus β^A determines the overall effort intensity, whereas the relative effort allocation across tasks is characterized by $\bar{\boldsymbol{\omega}}$. To exemplify the effort allocation under second-best, consider again the relation between two arbitrary activities e_i^* and e_j^*, $i \neq j$:

$$\frac{e_i^*}{e_j^*} = \frac{\bar{\omega}_i}{\bar{\omega}_j} \qquad i, j = 1, ..., n\ , \ \ i \neq j. \tag{2.15}$$

The agent places relatively more emphasis on tasks with a higher contribution to her performance evaluation in order to maximize the likelihood of obtaining the contracted bonus. From the principal's perspective, however, the motivated effort allocation is inefficient if it deviates from that implemented under first-best. Recall that the agent's effort allocation is distorted if there exists no constant $\lambda > 0$ satisfying $\boldsymbol{e}^* = \lambda \boldsymbol{e}^{fb}$. Substituting $\boldsymbol{e}^*$ and $\boldsymbol{e}^{fb}$ leads to $\bar{\boldsymbol{\omega}}\beta = \lambda\boldsymbol{\mu}$. Apparently, effort distortion is rooted in the misalignment of the agent's performance evaluation with respect to firm value. To put it differently, effort distortion occurs if the agent's performance evaluation is incongruent, i.e. does not perfectly capture her contribution to firm value. In contrast, if the principal has access to a congruent information system, the agent can be motivated to implement the non-distorted (first-best) effort allocation. Formally, the costless information system is congruent if there exists a constant $\xi > 0$ satisfying $\boldsymbol{\mu} = \xi\bar{\boldsymbol{\omega}}$. Otherwise, the agent's performance evaluation is incongruent and—when applied in an incentive contract—motivates her to implement an inefficient effort allocation across relevant tasks.

To exemplify the preceding observations, suppose the agent can implement two activities e_1 and e_2, where e_2 does not contribute to firm value ($\mu_2 = 0$). Nonetheless, e_2 is suitable to positively influence her performance evaluation ($\bar{\omega}_2 > 0$).[8] Accordingly, it would be desirable from the principal's perspective to exclude e_2 from the performance measurement. However, since the performance evaluation is non-separable in the different activities, the principal is compelled to accept the implementation of e_2. This suggests two inefficiencies: First, the principal eventually rewards non-valuable activities (e_2), and second, she needs to compensate the agent for the implementation of these activities in order to ensure her participation. However, even if e_2 contributes little to firm value relative to e_1 ($\mu_1 > \mu_2$) but the expected signal is relatively more sensitive in e_2 than in e_1 ($\bar{\omega}_1 < \bar{\omega}_2$), the induced effort allocation is distorted as the agent inefficiently places more emphasis on tasks 2 than on task 1.

Proposition 2.1 *The optimal bonus contract is characterized by* $\alpha^{A*} = 0$ *and*

$$\beta^{A*} = \frac{\boldsymbol{\mu}^t\bar{\boldsymbol{\omega}}}{2\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}}}. \tag{2.16}$$

The principal's expected second-best profit is

$$\Pi^* = \frac{(\boldsymbol{\mu}^t\bar{\boldsymbol{\omega}})^2}{4\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}}}. \tag{2.17}$$

Proof First observe that the principal needs to set $\beta^A > 0$ in order to ensure that the agent implements $e_i > 0$ for at least one $i \in \{1, ..., n\}$. As a consequence, (2.12) is satisfied as long as (2.13) holds and therefore omits. Recall that $\boldsymbol{e}^* = \beta^A\bar{\boldsymbol{\omega}}$. Thus, the Lagrangian becomes

$$\mathcal{L}(\alpha^A, \beta^A) = \boldsymbol{\mu}^t\bar{\boldsymbol{\omega}}\beta^A - \alpha^A - (\beta^A)^2\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}} + \lambda\left[\alpha^A + \frac{1}{2}(\beta^A)^2\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}}\right] + \xi\alpha^A. \tag{2.18}$$

The corresponding first-order conditions are

$$-1 + \lambda + \xi = 0, \tag{2.19}$$

$$\boldsymbol{\mu}^t\bar{\boldsymbol{\omega}} + \beta^A\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}}\,(\lambda - 2) = 0, \tag{2.20}$$

[8] Feltham and Xie [1994] analyze a similar example in a setting, where the agent is risk-averse. They refer to this particular phenomenon as *window dressing*.

and the complementary slackness conditions,

$$\lambda\left[\alpha^A + \frac{1}{2}(\beta^A)^2\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}}\right] = 0, \tag{2.21}$$

$$\xi\alpha^A = 0. \tag{2.22}$$

There are two cases to consider: (*i*) $\lambda > 0$, and (*ii*) $\lambda = 0$. First, suppose $\lambda > 0$. In this case, $\alpha^A + (\beta^A)^2\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}}/2 = 0$ due to (2.21). Since it is required that $\alpha^A \geq 0$, this would imply that $\alpha^{A*} = 0$, $\beta^{A*} = 0$, and consequently, $\boldsymbol{e}^* = (0, ..., 0)^t$. Therefore, $\lambda > 0$ cannot be a solution of this problem. Thus, $\lambda = 0$. Then, the second complementary slackness condition additionally implies that $\xi = 1$. Consequently, $\alpha^{A*} = 0$ due to (2.22). Solving (2.20) for β^A with $\lambda = 0$ leads to

$$\beta^{A*} = \frac{\boldsymbol{\mu}^t\bar{\boldsymbol{\omega}}}{2\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}}}. \tag{2.23}$$

Finally, the principal's expected profit can be obtained by substituting α^{A*} and β^{A*} in the principal's objective function.

Q.E.D.

Observe that the optimal bonus β^{A*}—and as a consequence the principal expected profit Π^*—depends on the relation between vector $\boldsymbol{\mu}$ and vector $\bar{\boldsymbol{\omega}}$. We can infer from preceding observations that the agent implements an inefficient effort allocation if $\boldsymbol{\mu}$ and $\bar{\boldsymbol{\omega}}$ are linearly independent.[9] Baker [2002] demonstrated that performance measure congruity can be characterized by the angle between these two vectors. To provide a measure of the induced effort distortion and the corresponding efficiency loss, I subsequently adapt Baker's geometric interpretation to the underlying setting with a risk-neutral and financially constrained agent. This eventually provides the analytical foundation for the subsequently analyzed costly improvement of the agent's performance evaluation aimed at motivating a more efficient effort allocation.

Lemma 2.1 *The angle $\bar{\varphi} \in [0, \pi/2]$ between vector $\boldsymbol{\mu}$ and vector $\bar{\boldsymbol{\omega}}$ measures the induced effort distortion and associated efficiency loss if the bonus contract is dependent on $\bar{S}$.*[10]

Proof See appendix.

This lemma indicates that the angle $\bar{\varphi}$ between the vector of the expected firm value sensitivities $\boldsymbol{\mu}$ and the vector of the expected signal sensitivities $\bar{\boldsymbol{\omega}}$ measures not only the congruity of a performance measure as emphasized by Baker [2002], but also the induced effort distortion and the corresponding efficiency loss.[11] The

[9] Formally, vector $\boldsymbol{\mu}$ and vector $\bar{\boldsymbol{\omega}}$ are linearly independent if there exists no constant $\lambda > 0$ satisfying $\boldsymbol{\mu} = \lambda\bar{\boldsymbol{\omega}}$.

[10] Throughout this essay, angles are represented in radian measures.

[11] The scaling of $\mathrm{E}[\bar{S}|\boldsymbol{e}]$ characterized by $\|\bar{\boldsymbol{\omega}}\|$ does not affect the efficiency of the bonus contract since the agent is risk-neutral. With risk-averse agents, however, the scaling is crucial since it affects the precision of performance measures and hence, the agent's required risk-premium, see e.g. Baker et al. [2002], Gibbons [2005], or chapter 1 for further discussions.

relation of these two vectors in the n-dimensional space is sufficient to characterize the inefficiency provoked by the application of an incongruent performance evaluation in the agent's incentive contract. Observe that the measure $\bar{\varphi}$ is negatively related to effort distortion and the corresponding efficiency loss. A smaller angle $\bar{\varphi}$ characterizes a more congruent information system, and therefore, induces less effort distortion. The provision of more congruent incentives is also desirable from the principal's perspective because it leads to a higher expected profit, as emphasized by lemma 2.1. To observe this, one can use the relations for vector products and re-write the principal's expected profit as

$$\Pi(\bar{\varphi}) = \frac{1}{4}\|\boldsymbol{\mu}\|^2 \cos^2 \bar{\varphi}, \tag{2.24}$$

where $\|\boldsymbol{\mu}\|$ is the length of vector $\boldsymbol{\mu}$. Observe that the principal's expected profit is decreasing in $\bar{\varphi}$. The rationale for this observation is that a less congruent performance evaluation (higher $\bar{\varphi}$) motivates the agent to implement a less efficient effort allocation. This entails two inefficiencies: (i) the expected firm value is less than it would have been under the implementation of less distorted effort; and (ii), the principal has to compensate the agent even for inefficiently chosen effort allocations in order to ensure her participation. Both effects deteriorate the efficiency of the bonus contract, and consequently, diminish the principal's expected profit.

Previous research dealing with single-task agency relations proposed several criteria to rank information systems with respect to their contract efficiency.[12] However, these ranking criteria are in general not applicable when the agent's effort is multi-dimensional. If the agent is risk-averse, I have shown in chapter 1 that the relative value of information systems is determined by their respective congruency relative to their precision. In this framework, however, the agent is risk-neutral and uncertainty in her performance evaluation does not effect the contract efficiency. Accordingly, an information system is weakly superior to any other information system from the principal's perspective if it motivates a weakly more efficient effort allocation, and consequently, results in a weakly higher expected profit.

Proposition 2.2 *Suppose there exists a non-empty set of information systems* $\mathbf{I}$ *with* $\bar{S}_i \in \{0, 1\}$, $i \in \mathbf{I}$. *Then, information system* k *generating* $\bar{S}_k$, $k \in \mathbf{I}$, *is weakly superior to any other information system if and only if,*

$$\bar{\varphi}_k \leq \bar{\varphi}_l, \quad \forall\, l \in \mathbf{I},\ l \neq k, \tag{2.25}$$

where $\bar{\varphi}_i$ *denotes the congruity measure for information system* $i \in \mathbf{I}$.

The emphasized measure $\bar{\varphi}$ for the congruity of an information system and the induced effort distortion additionally allows us to rank information systems in multi-task agencies if all involved parties are risk-neutral. The superior information system—when applied in an incentive contract—induces the least effort distortion, and therefore, minimizes the efficiency loss for the principal.

[12] For ranking of information systems in single-task agency relations with risk-averse agents, see e.g. Kim and Suh [1991]; and for a setting with a risk-neutral agent facing a liability limit constraint, see Demougin and Fluet [2001].

Next, consider the congruity of $\bar{S}$ and its effect on the agent's expected utility. Substituting $\boldsymbol{e}^*$ with the optimal bonus contract $(\alpha^{A*}, \beta^{A*})$ in (2.5) gives

$$U^A(\bar{\varphi}) = \frac{1}{8}\|\boldsymbol{\mu}\|^2 \cos^2 \bar{\varphi}. \tag{2.26}$$

Obviously, the congruity measure $\bar{\varphi}$ additionally influences the agent's expected utility. The rationale for this observation is that the optimal bonus contract reflects the congruity of $\bar{S}$ in order to adjust the provision of incentives appropriately.

Proposition 2.3 *The agent extracts a rent for all $\bar{\varphi} \in [0, \pi/2)$. The rent is maximized for a congruent expected signal with $\bar{\varphi} = 0$, and decreasing in $\bar{\varphi}$.*

Proof The first derivative of $U^A(\bar{\varphi})$ with respect to $\bar{\varphi}$ gives

$$\frac{\partial U^A(\bar{\varphi})}{\partial \bar{\varphi}} = -\frac{1}{4}\|\boldsymbol{\mu}\|^2 \sin \bar{\varphi} \cos \bar{\varphi}. \tag{2.27}$$

Since $2 \sin \bar{\varphi} \cos \bar{\varphi} = \sin(2\bar{\varphi})$, this is equivalent to

$$\frac{\partial U^A(\bar{\varphi})}{\partial \bar{\varphi}} = -\frac{1}{8}\|\boldsymbol{\mu}\|^2 \sin(2\bar{\varphi}), \tag{2.28}$$

which is strictly negative for all $\bar{\varphi} \in (0, \pi/2)$ due to the definition of the sine. Furthermore, $U^A(\bar{\varphi})$ is maximized for $\bar{\varphi} = 0$, which implies $U^A(0) = \|\boldsymbol{\mu}\|^2/8$. In contrast, $U^A(\pi/2) = 0$.

Q.E.D.

Since the agent's liability limit is always binding (see proof of proposition 2.1), the principal must leave her a rent. More interestingly, however, is the observation that the agent extracts a higher rent, the more congruent the underlying information system is. This can be observed because providing the agent with more congruent incentives (lower $\bar{\varphi}$) leads to the implementation of less distorted effort. Then, it is beneficial from the principal's perspective to enhance the bonus aimed at motivating a higher effort intensity. To see this, one can re-write the optimal bonus as

$$\beta^{A*}(\bar{\varphi}) = \frac{\|\boldsymbol{\mu}\|}{2\|\bar{\boldsymbol{\omega}}\|} \cos \bar{\varphi}. \tag{2.29}$$

First observe that $\beta^{A*}(\bar{\varphi})$ is increasing in the performance measure congruity, i.e. $\bar{\varphi}$ decreases. As a result of the agent's liability limit, enhancing $\beta^{A*}(\bar{\varphi})$ contemporaneously leads to a higher rent extraction. However, if $\bar{\varphi} = \pi/2$, the principal sets $\beta^{A*}(\pi/2) = 0$, thereby implying that the agent does not extract a rent. This observation is due to the fact that the application of $\bar{S}$ with $\bar{\varphi} = \pi/2$ in a bonus contract would motivate the agent to implement an effort allocation which does not contribute to firm value. To see this, note that $\text{Prob}\{V = 1|\boldsymbol{e}^*\} = \beta^A \|\boldsymbol{\mu}\| \|\bar{\boldsymbol{\omega}}\| \cos \bar{\varphi}$. Consequently, $\text{Prob}\{V = 1|\boldsymbol{e}^*\} = 0$ if $\bar{\varphi} = \pi/2$.

2.5 Costly Performance Measurement

As the observations in preceding sections indicate, the contract efficiency in multitask agencies is directly determined by the congruity of an available information system if all parties are risk-neutral. Accordingly, the principal is better off if she has access to an information system that better reflects the agent's contribution to firm value, and can therefore be utilized to generate a more efficient effort allocation.

Feltham and Xie [1994] and Datar et al. [2001] have shown that the principal can mitigate effort distortion by combing multiple performance measures appropriately. Since they restricted their analysis to the aggregation of costless available performance measures, it is intuitive to expand on their initial theoretical foundation to focus on the next logical step of investigating investment decisions with the objective of improving the congruity of an information system. Suppose the principal centrally invests in assets which are suitable to generate additional measures about the agent's performance. The principal can therefore improve the congruity of the information system by incorporating the additionally acquired measures appropriately. The more measures generated, the more congruent will be the eventually established information system as a representation of all available performance measures. The principal's investment into the improvement of the information system is henceforth referred to as *centralization*.

To exemplify the underlying idea of costly performance measurement, consider a worker who is employed for producing goods. Suppose these goods achieve a higher price on the market, the better their quality is. The previous analysis indicates that the worker's incentive contract should incorporate certain quality measures to ensure the achievement of a desired quality level. In contrast to the produced quantity, however, verifying the quality is costly. If the firm wants to maintain a certain quality standard, it is necessary to invest in a quality verification mechanism, e.g. by acquiring a machine suitable to verify whether the quality of produced items is within a specified tolerance level. Both performance measures—produced quantity and achievement of a desired quality—can be appropriately combined and used to provide the worker with incentives. Particularly, the firm can commit to pay only a bonus if the produced quantity of items satisfying a desired quality exceeds a predefined number. This incentive scheme eventually ensures that the worker implements a more efficient effort allocation. Roughly speaking, the worker is now motivated to place more emphasis on the achievement of a certain quality standard besides producing a sufficient quantity.

Consider the principal's investment into the improvement of the agent's performance evaluation aimed at providing her with more congruent incentives. Let $m \in \mathbb{R}^+$ be the measurement intensity, where a higher intensity generates a more congruent information system. The principal commits to a certain intensity m by investing in the measurement system prior to negotiating with the agent about her incentive contract.

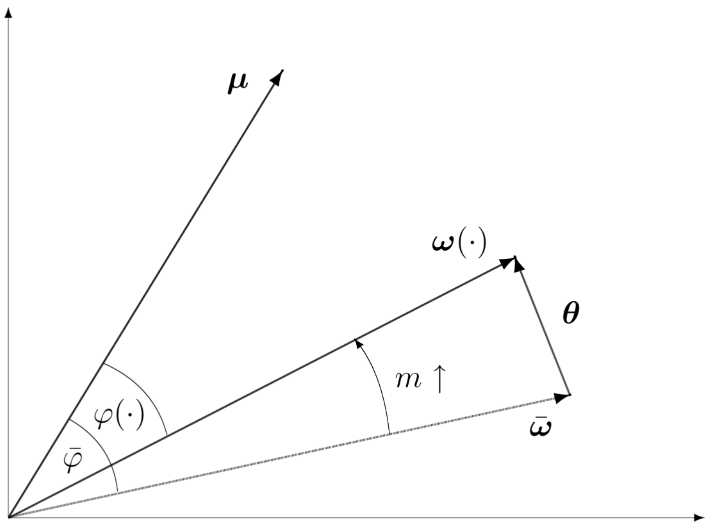

Figure 2.1: The Measurement Technology

Assumption 2.1 *The implementation of* $m \in \mathbb{R}^+$ *generates a new verifiable and binary statistic* $S(m) \in \{0, 1\}$ *characterized by*

(i): $\text{Prob}\{S = 1|m, \bar{\varphi}, \boldsymbol{e}\} = \min\{\boldsymbol{\omega}(\varphi(m, \bar{\varphi}))^t\boldsymbol{e}, 1\},$
(ii): $\varphi(m, \bar{\varphi}) < \bar{\varphi} \quad \forall\, m > 0.$

The first condition emphasizes that the implemented measurement intensity m generates a new information system which is represented by the binary statistic $S(m) \in \{0, 1\}$. The corresponding expected signal is further characterized by the sensitivity vector $\boldsymbol{\omega}(\varphi(m, \bar{\varphi}))$. Its relation to the sensitivity vector $\boldsymbol{\mu}$ of the expected firm value is thereby determined by the angle $\varphi(\cdot)$, which measures the congruity of $S(m)$. The angle $\varphi(m, \bar{\varphi})$ as the congruity measure is a function of the implemented measurement intensity m and the congruity measure $\bar{\varphi}$ of the costless available information system $\bar{S}$. The final condition ensures that implementing a strictly positive intensity m leads eventually to a more congruent information system.[13]

The underlying measurement technology is illustrated in figure 2.1. The more intense the costly information acquisition m is, the smaller becomes the angle between the vector of the expected firm value sensitivities $\boldsymbol{\mu}$ and the new vector of the expected signal sensitivities $\boldsymbol{\omega}(\cdot)$. Note that $\boldsymbol{\omega}(\cdot)$ is not necessarily on the plane spanned by $\boldsymbol{\mu}$ and $\bar{\boldsymbol{\omega}}$. It is only required that $\boldsymbol{\omega}(\cdot)$ is characterized by a smaller angle to $\boldsymbol{\mu}$ than vector $\bar{\boldsymbol{\omega}}$, i.e $\varphi(m, \bar{\varphi}) < \bar{\varphi}$. Observe further that geometrically the new vector $\boldsymbol{\omega}(\cdot)$ is the sum of vector $\bar{\boldsymbol{\omega}}$ and vector $\boldsymbol{\theta}$, where $\boldsymbol{\theta}$ represents all information additionally acquired through costly performance measurement.

Recall that the optimal bonus β^{A*} is normalized by $\|\bar{\boldsymbol{\omega}}\|$ in order to exclude potential effects by different lengths of $\bar{\boldsymbol{\omega}}$. The same will be true if the length of $\boldsymbol{\omega}(\cdot)$ varies with m. For parsimony purposes and without loss of generality, I normalize the length of every new generated vector $\boldsymbol{\omega}(\cdot)$ to $\|\boldsymbol{\omega}(\cdot)\| = \|\bar{\boldsymbol{\omega}}\|$.[14]

[13]Technically, the last condition requires that new generated performance measures are not sufficient statistics of other available measures.

[14]This assumption additionally ensures interior solutions, see section 2.2.

Assumption 2.2 *The generated information system with $\varphi(m, \bar{\varphi})$ as its congruity measure is characterized by*

$$\begin{array}{ll} \textit{(i):} & \varphi_m < 0 \textit{ and } \varphi_{mm} > 0 \;\forall\, m \geq 0, \\ \textit{(ii):} & \varphi_{\bar{\varphi}} > 0 \textit{ and } \varphi_{m\bar{\varphi}} < 0 \;\forall\, m \geq 0, \\ \textit{(iii):} & \varphi\left(0, \bar{\varphi}\right) = \bar{\varphi} \textit{ and } \varphi\left(m, 0\right) = 0, \\ \textit{(iv):} & \bar{\varphi} \in [0, \pi/4], \end{array}$$

where φ_i denotes the first, and φ_{ii} the second derivative of $\varphi(\cdot)$ with respect to i, $i = m, \bar{\varphi}$.[15]

Condition (i) implies that a higher measurement intensity m enhances the congruity of the generated information system (smaller $\varphi(\cdot)$), whereas the marginal effect of reducing $\varphi(\cdot)$ is decreasing in m. Condition (ii) states that the new generated information system $S(m)$ is less congruent for a given measurement intensity m, the less congruent the costless information system $\bar{S}$ is. Moreover, the marginal effect of reducing $\varphi(\cdot)$ by implementing an arbitrary intensity m is higher, the less congruent the costless information system is, i.e. the higher $\bar{\varphi}$ is. Condition (iii) emphasizes that without costly performance measurement, only the costless information system is available. Additionally, a perfectly congruent information system cannot be improved. To understand the last condition, observe from (2.24) that the expected gross benefit contingent on an arbitrary measurement intensity m becomes

$$\widehat{V}(m) = \frac{1}{4} \|\boldsymbol{\mu}\|^2 \cos^2 \varphi\left(m, \bar{\varphi}\right). \tag{2.30}$$

It can be verified that $\widehat{V}(m)$ is strictly concave increasing in m for $\varphi(\cdot) \in (0, \pi/4)$, whereas the shape of $\widehat{V}(m)$ is ambiguous for $\varphi(\cdot) \in [\pi/4, \pi/2)$ and depends on the particular behavior of $\varphi(\cdot)$ in m. Consequently, condition (iv) guarantees that the first-order approach is sufficient for identifying the optimal measurement intensity.

Assumption 2.3 *The implementation of $m \in \mathbb{R}^+$ imposes costs $C(m)$ characterized by*

$$\begin{array}{ll} \textit{(i):} & C'(m) > 0,\; C''(m) > 0, \textit{ and } C'''(m) \geq 0, \\ \textit{(ii):} & C(0) = C'(0) = 0, \\ \textit{(iii):} & \lim_{m \to \infty} C'(m) = \infty. \end{array}$$

The first condition emphasizes that the investment costs for improving the agent's performance evaluation are strictly convex increasing in the conducted measurement intensity. The assumption about the third derivative is for technical reasons only. Condition (*ii*) states that the principal bears no additional costs when she does not improve the information system. The final condition ensures an interior solution since the generation of a perfectly congruent information system is prohibitively costly.[16]

[15] For parsimony purposes, I suppress the arguments for the respective derivative, unless it is necessary for specific comparisons.

[16] This condition implicitly requires that the quantity of relevant tasks is strictly greater than the number of available performance measures that provide different information about the agent's effort allocation. For a formal analysis refer to chapter 1.

As explained earlier, the principal commits to a particular measurement intensity m by investing $C(m)$ prior to negotiating with the agent about her bonus contracts. As a consequence, the agent's bonus contract is now on the basis of $S(m)$ and therefore, depends on $\varphi(m, \bar{\varphi})$, but everything else remains identical. Hence, we can directly turn to the principal's problem for identifying the optimal measurement intensity m^*. The adjustment of (2.24) with respect to the introduced measurement technology implies that the optimal intensity m^* solves

$$\max_{m} \ \Pi^C(m, \bar{\varphi}) = \frac{1}{4}\|\boldsymbol{\mu}\|^2 \cos^2 \varphi\left(m, \bar{\varphi}\right) - C(m). \tag{2.31}$$

Since $2 \sin \varphi(\cdot) \cos \varphi(\cdot) = \sin\left(2\varphi(\cdot)\right)$, the first-order condition gives

$$\frac{1}{4}\|\boldsymbol{\mu}\|^2 \sin\left(2\varphi\left(m^*, \bar{\varphi}\right)\right)\left(-\varphi_m\right) = C'(m^*). \tag{2.32}$$

The principal enhances the measurement intensity m until the expected marginal gross benefit is equal to marginal costs. Since the expected gross benefit is concave, and investment costs are convex increasing in m, the first-order approach is also sufficient.[17] Observe that the optimal measurement intensity m^* depends implicitly on the congruity measure $\bar{\varphi}$ of the costless information system $\bar{S}$. The effect of $\bar{\varphi}$ on the optimal measurement intensity m^* and the principal's expected profit $\Pi^C(\cdot)$ is clarified by the subsequent proposition.

Proposition 2.4 *The optimal measurement intensity $m^*(\bar{\varphi})$ is increasing in $\bar{\varphi}$. Overall, the principal's expected profit $\Pi^C(m^*(\bar{\varphi}), \bar{\varphi})$ decreases in $\bar{\varphi}$.*

Proof See appendix.

The less congruent the costless available information system $\bar{S}$ is (higher $\bar{\varphi}$), the more distorted would be the motivated effort allocation if the principal uses $\bar{S}$ to provide the agent with incentives. Since this diminishes the principal's expected profit, it is optimal from her perspective to invest more in the improvement of the information system aimed at mitigating effort distortion. However, improving the information system imposes strictly convex increasing costs. Consequently, the additional inefficiency due to an increases of $\bar{\varphi}$ cannot be compensated by enhancing $m^*(\cdot)$. As a result, $\Pi^C(\cdot)$ decreases in $\bar{\varphi}$.

By committing to the optimal measurement intensity $m^*(\cdot)$, the principal ensures that the agent's performance evaluation is eventually characterized by a desired congruity measure $\varphi(m^*(\cdot), \bar{\varphi})$. This in turn motivates the agent to choose a

[17] To prove this analytically, observe that the second-order condition gives

$$\frac{\partial^2 \Pi^C(\cdot)}{\partial m^2} = \frac{1}{4}\|\boldsymbol{\mu}\|^2 \left[2\cos(2\varphi(\cdot))\varphi_m(-\varphi_m) + \sin\left(2\varphi\left(\cdot\right)\right)\left(-\varphi_{mm}\right)\right] - C''(m), \tag{2.33}$$

which is equivalent to

$$\frac{\partial^2 \Pi^C(\cdot)}{\partial m^2} = -\frac{1}{4}\|\boldsymbol{\mu}\|^2 \left[2\cos(2\varphi(\cdot))(\varphi_m)^2 + \sin\left(2\varphi\left(\cdot\right)\right)\varphi_{mm}\right] - C''(m). \tag{2.34}$$

Since $\cos(2\varphi(\cdot)) > 0$ and $\sin\left(2\varphi\left(\cdot\right)\right) > 0$ for all $\varphi(\cdot) \in (0, \pi/4)$, the second derivative is strictly negative. Hence, the first-order approach is sufficient.

more efficient effort allocation in comparison to the application of $\bar{S}$. The preceding analysis indicates that the agent's bonus contract reflects the congruity of her performance evaluation. To illustrate the effect of improving the incentive congruity on the agent's expected utility, observe that $U^A(\cdot)$ changes to

$$U^A(m^*(\bar{\varphi}), \bar{\varphi}) = \frac{1}{8}\|\boldsymbol{\mu}\|^2 \cos^2 \varphi(m^*(\bar{\varphi}), \bar{\varphi}). \tag{2.35}$$

Recall that $2\sin(\cdot)\cos(\cdot) = \sin(2(\cdot))$. Consequently, the first derivative of $U^A(\cdot)$ with respect to m is

$$\frac{\partial U^A(\cdot)}{\partial m} = \frac{1}{8}\|\boldsymbol{\mu}\|^2 \sin(2\varphi(\cdot))(-\varphi_m), \tag{2.36}$$

which is strictly positive for all $\varphi(\cdot) \in (0, \pi/2)$ since $\varphi_m(\cdot) < 0$. Accordingly, improving the information system contemporaneously implies a higher rent extraction by the agent. This in turn constitutes indirect costs for mitigating effort distortion if the agent faces a liability limit constraint.

2.6 Delegation

Instead of centrally investing in the improvement of the agent's performance evaluation, the principal can alternatively delegate the information acquisition to a supervisor. This can be favored by the principal if the supervisor is able to measure the agent's performance more efficiently than a centralized regime. However, employing a supervisor with the objective of improving the information system imposes a second moral hazard problem. Thus, the principal needs to provide the supervisor with appropriate incentives in order to ensure the implementation of a desired measurement intensity.

Suppose the principal recruits a risk-neutral supervisor who also faces a liability limit constraint. For parsimony, her reservation utility is normalized to zero. The supervisor can conduct a performance measurement with the intensity $m^S \in \mathbb{R}^+$ satisfying assumptions 2.1 and 2.2. A strictly positive measurement intensity therefore generates additional measures about the agent's performance, which can be used to enhance the congruence of her performance evaluation. The supervisor's performance measurement is therefore objective rather than subjective.[18] Recall the quantity/quality example from section 2.5. Instead of purchasing a machine verifying the achievement of a quality standard, the supervisor may verify and document the quality characteristics of produced items. Both information—the produced quantity and achieved quality—can be appropriately combined to provide the agent with more congruent incentives. Finally, the manipulation of generated performance measures is assumed to be prohibitively costly for the supervisor.

For the sake of comparability, the supervisor's disutility of conducting an arbitrary measurement intensity m^S is $C^S(m^S) = \eta C(m^S)$, where $C(m^S)$ is identical to the principal's investment costs for all $m^S = m$. Potential differences in the measurement efficiency are characterized by $\eta \in \mathbb{R}^+$. The ratio $1/\eta$ thereby measures the

[18]Subjective evaluations imply additional problems since implicit promises to reward favorable behavior needs to be self-enforcing in repeated games [Baker et al., 1994a], or induce favoritism by the evaluator [Prendergast and Topel, 1996].

supervisor's comparative advantage in generating the same performance measures relative to the principal's central investment.

The principal observes the realization of the binary statistic $S(m^S) \in \{0,1\}$ but cannot directly contract over m^S. Suppose the unfavorable signal $S(m^S) = 0$ is realized. This can occur because either the agent indeed failed to meet her performance objective, or the supervisor did not generate the required information such that there exists no evidence about the implementation of the desired effort allocation. Consequently, the principal needs to provide the supervisor with incentives to motivate a desired measurement intensity $m^S > 0$. The principal, however, receives several biased information about the conducted measurement intensity, which are summarized by the binary statistic $M \in \{0,1\}$.[19]

Assumption 2.4 *Let* $\Pr[M = 1|m^S] = \rho(m^S) \in [0,1)$*, where the probability* $\rho(m^S)$ *to realize the favorable signal* $M = 1$ *satisfies*

(i): $\rho(0) = 0$,
(ii): $\rho(m^S)$ *is twice-continuously differentiable with* $\rho'(m^S) > 0$ *and* $\rho''(m^S) \leq 0 \ \forall\, m^S \geq 0$,
(iii): $\lim_{m^S \to \infty} \rho(m^S) = 1$ *and* $\lim_{m^S \to 0} \rho'(m^S) = \infty$.

Condition (i) emphasizes that without improving the information system, the binary statistic M can never be favorable. The second and third conditions are standard and guarantee an interior and unique solution for the optimal measurement intensity.

In order to induce a desired measurement intensity, the principal provides the supervisor with a bonus contract w^S on the basis of the verifiable statistic M. In particular, let

$$w^S = \begin{cases} \alpha^S + \beta^S, & \text{if} \quad M = 1, \\ \alpha^S, & \text{if} \quad M = 0, \end{cases} \tag{2.37}$$

where α^S denotes the fixed transfer and β^S the bonus payment. The supervisor can increase the probability $\rho(m^S)$ to realize the favorable signal $M = 1$, and consequently, the likelihood to obtain the contracted bonus β^S, when she enhances her measurement intensity m^S. Since the supervisor is risk-neutral, the bonus contract provides her with the expected utility

$$U^S(m^S) = \alpha^S + \rho(m^S)\beta^S - \eta C(m^S). \tag{2.38}$$

Before deriving the optimal bonus contract for the supervisor, it is essential to briefly consider the timing of this problem. First, the principal offers the supervisor a bonus contract w^S. If she accepts the contract and participates, it provides her with incentives to implement a desired measurement intensity m^{S*}. Afterwards, the principal provides the agent with a bonus contract on the basis of her anticipated performance evaluation represented by the binary statistic $S(m^{S*})$. Then, the agent implements effort and the supervisor subsequently generates the additional performance measure(s). Afterwards, the agent's and supervisor's performance evaluations $S(m^{S*})$

[19] See Demougin and Fluet [2001] for a formal derivation that verifiable information about one-dimensional effort can be summarized by a binary statistic if parties are risk-neutral.

and M are realized and become public knowledge. Finally, all contracted payments take place.

The agent's bonus contract depends now on the supervisor's induced measurement intensity m^{S*}, but remains qualitatively the same as in section 2.5. Therefore, we can directly turn to the derivation of the supervisor's contract, which is aimed at maximizing the principal's expected profit while guaranteeing the supervisor's participation. Formally, the supervisor's optimal bonus contract solves

$$\max_{\alpha^S,\beta^S,m^S} \Pi^D \equiv \frac{1}{4}\|\boldsymbol{\mu}\|^2 \cos^2\varphi\left(m^S,\bar{\varphi}\right) - \alpha^S - \rho(m^S)\beta^S \tag{2.39}$$

s.t.

$$\alpha^S + \rho(m^S)\beta^S - \eta C(m^S) \geq 0 \tag{2.40}$$

$$m^S = \arg\max_{\tilde{m}^S} \alpha^S + \rho(\tilde{m}^S)\beta^S - \eta C(\tilde{m}^S) \tag{2.41}$$

$$\alpha^S + \beta^S \geq 0 \tag{2.42}$$

$$\alpha^S \geq 0. \tag{2.43}$$

Condition (2.40) is the supervisor's participation constraint and guarantees that it is in her interest to enter into this relationship. Additionally, (2.41) is the supervisor's incentive constraint. Finally, conditions (2.42) and (2.43) ensure that the bonus contract is compatible with the supervisor's liability limit.

Proposition 2.5 *The supervisor's optimal bonus contract is characterized by the fixed transfer $\alpha^{S*} = 0$ and the expected bonus*

$$B(m^{S*},\eta) = \frac{\eta C'(m^{S*})\rho(m^{S*})}{\rho'(m^{S*})}. \tag{2.44}$$

The optimal measurement intensity m^{S*} thereby solves

$$\frac{1}{4}\|\boldsymbol{\mu}\|^2 \sin\left(2\varphi\left(m^{S*},\bar{\varphi}\right)\right)(-\varphi_m) = \frac{\partial B(m^{S*},\eta)}{\partial m^S}. \tag{2.45}$$

Then, the principal receives

$$\Pi^D(m^{S*},\bar{\varphi},\eta) = \frac{1}{4}\|\boldsymbol{\mu}\|^2 \cos^2\varphi\left(m^{S*},\bar{\varphi}\right) - B(m^{S*},\eta). \tag{2.46}$$

Proof See appendix.

Consider first the supervisor's expected bonus $B(m^{S*},\eta)$ which comprises the optimal alignment to induce m^{S*}. Observe that the expected bonus is characterized by the likelihood ratio $\rho'(m^S)/\rho(m^S)$ which, according to Holmström [1979], measures the principal's propensity to expect that the supervisor has not implemented the anticipated measurement intensity m^{S*} when the favorable signal $M = 1$ is realized. The likelihood ratio therefore measures the precision of the supervisor's performance evaluation. Finally observe that $B(m^{S*},\eta)$ consists of the supervisor's relative measurement efficiency parameterized by η. The less efficient the supervisor's information acquisition is, characterized by a higher η, the higher must be the expected bonus payment $B(\cdot)$ in order to motivate the implementation of an arbitrary measurement intensity.

Proposition 2.6 *The supervisor extracts a rent for $m^{S*} > 0$, which is increasing in m^{S*}.*

Proof The supervisor obtains a rent if $R(m^{S*}, \eta) \equiv B(m^{S*}, \eta) - \eta C(m^{S*}) > 0$. Suppose first that $m^S = 0$. Since $\rho(0) = 0$, this implies that $B(0, \eta) = 0$, and consequently, $R(0, \eta) = 0$. In order to demonstrate that $R(m^{S*}, \eta) > 0$ for $m^{S*} > 0$, we can use the first derivative of $R(\cdot)$ with respect to m^S, which leads after re-arranging to

$$\frac{\partial R(\cdot)}{\partial m^S} = \frac{\rho(m^S)\eta C''(m^S)}{\rho'(m^S)} + \frac{\rho(m^S)\eta C'(m^S)(-\rho''(m^S))}{[\rho'(m^S)]^2}. \tag{2.47}$$

Observe that $\partial R(\cdot)/\partial m^S > 0$ for all $m^S > 0$. Since $R(0, \eta) = 0$ and $\partial R(\cdot)/\partial m^S > 0$, it follows that $R(\cdot) > 0$ for all $m^{S*} > 0$.

Q.E.D.

Since her liability limit constraint (2.43) is binding, the supervisor extracts a rent. This is because the principal cannot impose negative transfers to expropriate the supervisor's rent. However, despite the supervisor's rent extraction, it cannot be inferred that delegation is always less beneficial from the principal's perspective than a centralized investment. In particular, this depends on the supervisor's cost parameter η, which characterizes her comparative advantage in acquiring the desired measures about the agent's performance. If this relative advantage is sufficiently high (low η), the expected bonus for inducing an arbitrary measurement intensity can be eventually less than the principal's required investment. In contrast, employing a supervisor with only a slight comparative advantage can impose higher costs for an arbitrary measurement intensity due to her rent extraction.

Next, consider the optimality condition for m^{S*} emphasized by proposition 2.5. The optimal measurement intensity m^{S*} implicitly depends on two parameters: (i) the congruity measure $\bar{\varphi}$ of the costless information system $\bar{S}$; and (ii), the supervisor's relative measurement efficiency η. Since both parameters determine the optimal measurement intensity m^{S*}, they implicitly affect the congruity of the agent's performance evaluation, and consequently, the efficiency of her effort allocation across tasks.

Proposition 2.7 *The optimal measurement intensity $m^{S*}(\bar{\varphi}, \eta)$ is increasing in $\bar{\varphi}$ and decreasing in η. Overall, the principal's expected profit $\Pi^D(m^{S*}(\bar{\varphi}, \eta), \bar{\varphi}, \eta)$ is decreasing in $\bar{\varphi}$ and in η.*

Proof See appendix.

The first part of this result is obvious because a less congruent costless information system induces more distortion in the agent's effort allocation. In order to restrict this inefficiency, it is optimal to provide the supervisor with more powerful incentives aimed at motivating the implementation of a higher measurement intensity m^{S*}. Note that this directly leads to a higher rent extraction by the supervisor and by the agent. However, improving the congruity of the information system imposes convex increasing costs $B(\cdot)$. Thus, the additional inefficiency of an increasing

$\bar{\varphi}$ cannot be perfectly compensated by inducing a higher intensity $m^{S*}(\cdot)$. As a consequence, $\Pi^D(\cdot)$ decreases in $\bar{\varphi}$. Moreover, it is more costly for the principal to motivate an arbitrary measurement intensity if the supervisor provides a less relative measurement efficiency (higher η). In this case, it is optimal to induce a lower measurement intensity, contemporaneously implying that the principal receives a lower expected profit.

2.7 When is Delegation Profitable?

According to previous observations, one crucial factor for the principal's preference in term of centralizing or delegating the information acquisition is the supervisor's relative measurement efficiency η. If η is sufficiently low, one can expect that the principal favors delegation, and vice versa. However, it is not obvious how the congruity of the costless information system $\bar{\varphi}$ affects the profitability of delegation relative to a centralized investment. The purpose of this section is therefore to investigate, how the relationship between the congruity of the costless information system $\bar{\varphi}$ and the supervisor's relative measurement efficiency η affect the optimal organizational design.

Before turning to the principal's preference for a particular organizational design aimed at efficiently improving the information system, it is first necessary to compare the optimal measurement intensities for both considered alternatives. Intuitively, one could expect that the supervisor establishes a more congruent information system if delegation is more profitable than centralization. Note, however, this is not in general true. It particularly depends on the shape of the supervisor's expected bonus $B(\cdot)$ characterized by the inverted likelihood ratio $\rho(m^S)/\rho'(m^S)$. To identify whether or not $m^{S*}(\cdot) > m^*(\cdot)$, suppose the supervisor would implement the same measurement intensity as the principal does in the optimum, i.e. $m^S = m^*(\cdot)$. Then, the supervisor's optimal measurement intensity $m^{S*}(\cdot)$ is higher than the principal's optimal intensity $m^*(\cdot)$ if her expected bonus is less increasing in the point $m^S = m^*(\cdot)$ than the principal's investment costs. Formally, $m^{S*}(\cdot) > m^*(\cdot)$ if

$$\frac{\partial B(m^*(\cdot), \eta)}{\partial m^S} < C'(m^*(\cdot)). \tag{2.48}$$

I demonstrate in the appendix that this is equivalent to

$$\frac{\rho(m^*)}{\rho'(m^*)}\left[\frac{C''(m^*)}{C'(m^*)} - \frac{\rho''(m^*)}{\rho'(m^*)}\right] < \frac{1-\eta}{\eta}, \tag{2.49}$$

where $m^* \equiv m^*(\cdot)$. If the inverted likelihood ratio $\rho(\cdot)/\rho'(\cdot)$ is sufficiently low in m^* and the supervisor provides an adequate relative measurement efficiency (low η), it is optimal from the principal's perspective to motivate a higher measurement intensity under delegation than it would be optimal for a centralized investment. In this case, delegation leads to the provision of more congruent incentives than centralization, thereby motivating the agent to implement less distorted effort. In general, the principal's decision on whether to centralize or to delegate the information acquisition depends, besides the precision of the supervisor's performance evaluation

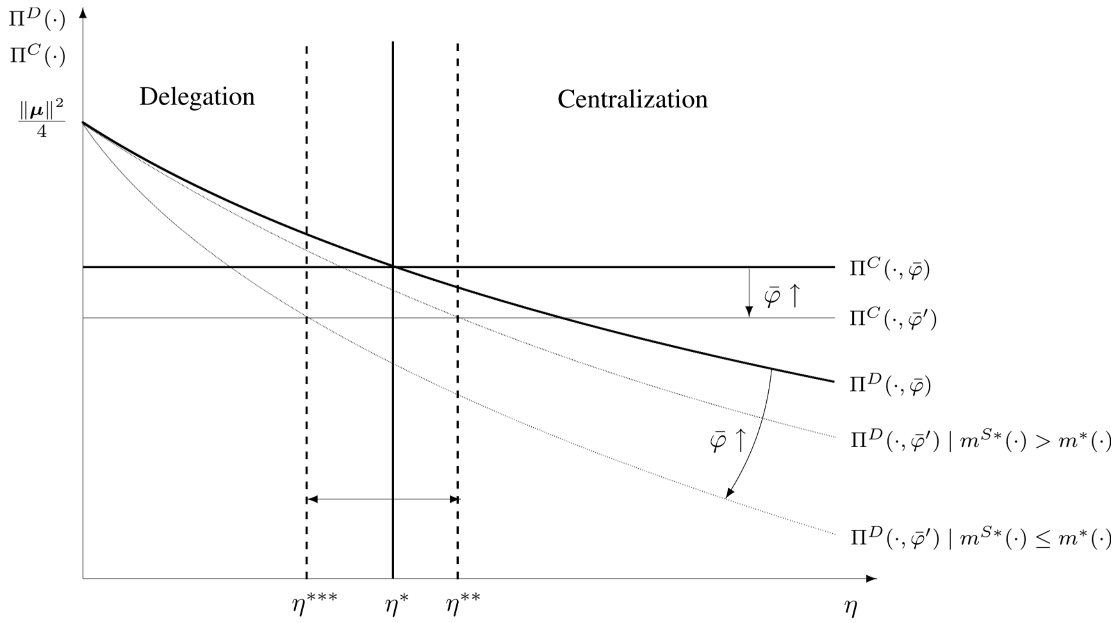

Figure 2.2: The Optimal Organizational Design

$\rho(m^S)/\rho'(m^S)$, on the congruity of the costless information system $\bar{\varphi}$, and on the supervisor's relative measurement efficiency η.

Proposition 2.8 *There exists a cut off cost parameter $\eta^* \in (0,1)$ implying that the principal is indifferent between centralization and delegation. Moreover, η^* is strictly increasing (decreasing) in $\bar{\varphi}$ if $m^{S*}(\cdot) > (\leq) m^*(\cdot)$.*

Proof See appendix.

Delegating the information acquisition to the supervisor is only favored by the principal if the supervisor provides a sufficient comparative advantage in measuring the agent's performance, which countervails her rent extraction. More interestingly, however, is the observation that the supervisor's minimum required measurement efficiency is lower (higher η^*), the less congruent the costless information system is (higher $\bar{\varphi}$). This applies when the supervisor's performance evaluation is sufficiently precise such that delegation leads to a higher measurement intensity than centralization. If in contrast the supervisor's performance evaluation is sufficiently imprecise such that $m^{S*}(\cdot) \leq m^*(\cdot)$, the reversed observation obtains.

The preceding observations are illustrated in figure 2.2, where the supervisor's relative measurement efficiency η is depicted on the horizontal axis, and the principal expected profits for centralization and delegation are represented on the vertical axis. Since $\Pi^C(\cdot)$ is independent of η, its curve is parallel to the horizontal axis, whereas $\Pi^D(\cdot)$ is decreasing in η, see proposition 2.7. The intersection of both curves characterizes the cut off η^*, where the principal is indifferent between centralization and delegation. For every $\eta < \eta^*$, the principal delegates the information acquisition to the supervisor, and centralizes, otherwise. Now suppose that the costless information system becomes more incongruent, i.e. $\bar{\varphi}$ increases to $\bar{\varphi}'$. As a result, it is optimal to conduct a higher measurement intensity under centralization and delegation in order to mitigate the agent's effort distortion, see proposition 2.4 and

2.7. However, enhancing the measurement intensity imposes convex increasing costs, thereby implying that $\Pi^C(\cdot)$ and $\Pi^D(\cdot)$ decrease. Observe that the decline of $\Pi^D(\cdot)$ in $\bar{\varphi}$ particularly depends on the value of η, and on whether or not $m^{S*}(\cdot) > m^*(\cdot)$. Suppose for a moment that $m^{S*}(\cdot) > m^*(\cdot)$. If $\eta \leq \eta^*$, the marginal costs for enhancing the measurement intensity are lower for delegation than for centralization. As a consequence, $\Pi^D(\cdot)$ is less decreasing in $\bar{\varphi}$ than $\Pi^C(\cdot)$ for $\eta \leq \eta^*$. Graphically, this implies that η^* increases to η^{**}. Generally speaking, the supervisor's minimum required relative measurement efficiency for delegation to be superior, is lower, the less congruent the costless information system is. In contrast, a more congruent information system imposes higher requirements on the supervisor's relative measurement efficiency for delegation to be favored by the principal. However, if $m^{S*}(\cdot) \leq m^*(\cdot)$, the reverse is true such that η^* decreases to η^{***}.

To illustrate the relevance of the preceding observations for the design of organizations, consider a firm with two departments A and B, where the costless information system about employees' effort allocation is assumed to be more congruent in department A than in B. For example, department A can be the sales department and B the human resources department. For simplicity, assume that there is only one costless performance measure in each department available: (*i*) the achieved sales in the sales department; and (*ii*), the office-hours in the human resources department. The achieved sales can thereby be inferred to be a more congruent performance measure than the office-hours in the human resources department. Suppose the supervisor's performance evaluation is sufficiently precise in both departments, thereby implying $m^{S*}(\cdot) > m^*(\cdot)$. The reversed argumentation applies for $m^{S*}(\cdot) \leq m^*(\cdot)$. According to previous results, it is optimal to conduct a higher measurement intensity in department B than in department A aimed at improving the respective information system and hence, mitigating effort distortion. Furthermore, the minimum required measurement efficiency a potential supervisor has to provide, is higher for department A than for department B. This eventually leads to the conclusion that delegation for department B (human resources department) is more likely than for department A (sales department) since potential supervisors presumably meet the requirements with respect to their measurement efficiency for department B, rather than for department A.

The previous analysis reveals that job characteristics, particulary the congruity of costless performance measures, determine how costly performance measurement will be organized. Particularly, if the supervisor's performance evaluation is adequately precise, the previous analysis provides two crucial implications to explain why we can observe different practises for costly performance measurement even within the same organization. First, a sufficiently incongruent costless information system generally leads to delegation as the superior alternative. The rationale for this observation is that delegation—to be preferred by the principal—requires only a low relative measurement efficiency which is more likely to be provided by a potential supervisor. Second, the more congruent the costless information system is, the more likely is a centralized investment in the information acquisition. This can be observed because delegation—in order to be the more profitable alternative—would impose high requirements on the supervisor's relative measurement efficiency, which is less likely to be achieved by a potential supervisor. The reverse inference applies if the supervisor's performance evaluation is sufficiently imprecise.

2.8 Conclusion

In economic relationships that are subject to moral hazard, objective performance measurement is frequently utilized to provide agents with incentives. However, applying performance measures in incentive contracts may motivate agents to implement inefficient effort allocations across relevant tasks. This occurs if an available performance evaluation does not perfectly reflect the agent's contribution to firm value. Then, it is desirable from the perspective of firms to use mechanisms which are suitable to mitigate this inefficiency. This essay investigates the costly acquisition of additional performance measures as one mechanism to alleviate effort distortion. The main emphasis is on the principal's preference for either centrally investing in assets that can be utilized to measure the agent's performance, or delegating the acquisition of additional measures to a supervisor.

This essay demonstrates that the less congruent the costless information system is, the greater is the investment in its improvement aimed at restricting effort distortion. This applies for both considered alternatives, i.e. for the centralized investment and for the delegation of the performance measurement to the supervisor. However, enhancing the congruity of the agent's performance evaluation does not only improve her effort allocation, but also provides her with a higher rent due to her liability limit.

The analysis indicates that employing a financially constrained supervisor for measuring the agent's performance can only be beneficial if she provides a sufficient relative measurement efficiency, which countervails her rent extraction. This essay illustrates that the profitability of delegation depends on three factors: (i) the precision of the supervisor's evaluation system, (ii) the supervisor's comparative cost advantage in obtaining the required information; and (iii), the congruence of the costless available information system about the agent's effort. Particularly, if the supervisor's performance evaluation is sufficiently precise, this investigation leads to the subsequent two implications. First, the principal generally prefers to delegate the information acquisition to the supervisor if the costless information system is sufficiently incongruent. This is because a less congruent costless information system imposes lower requirements on the supervisor's relative measurement efficiency, which is more likely to be satisfied by a potential supervisor. Second, the principal favors in general a centralized investment if the costless information system is sufficiently congruent. This can be observed because delegation—in order to be the superior strategy—imposes a high requirement on the supervisor's relative measurement efficiency, which is less likely to be achievable by a potential supervisor. The reverse can be observed if the supervisor's performance evaluation is sufficiently imprecise.

This essay contributes to previous multi-task agency literature by shedding light on two potential alternatives for the costly generation of additional measures about the agent's effort aimed at improving the efficiency of her effort allocation. However, our understanding of how organizations respond to induced effort distortion is far from complete. There are several issues which are promising to consider in more detail. First, there exists empirical evidence that non-linear compensation schemes provoke individuals not only to distort their effort among different tasks, but also to distort their effort over time.[20] Accordingly, besides motivating an efficient effort intensity and allocation across tasks, the additional objective of performance measurement is to motivate an efficient effort allocation over time. Furthermore, this essay focussed on the improvement of an information system in a setting with risk-neutral parties. The next logical step is to elaborate on the information acquisition in a setting with a risk-averse agent. In this case, the purpose of acquiring additional performance measures is not only restricted to improve her effort allocation, but also to reduce incentive risk. By incorporating a risk-averse agent in the underlying framework, prospective research may focus on this additional trade-off. However, I leave these important issues to future research.

[20] See e.g. Asch [1990] and Oyer [1998].

2.9 Appendix

Proof of Lemma 2.1.
As emphasized, effort distortion refers to the relation of $\boldsymbol{e}^*$ to $\boldsymbol{\mu}$. Hence, it can be characterized by the vector product $\boldsymbol{\mu}^t\boldsymbol{e}^* = \beta^A\boldsymbol{\mu}^t\bar{\boldsymbol{\omega}}$. Applying the relation for vector products gives $\boldsymbol{\mu}^t\boldsymbol{e}^* = \beta^A\|\boldsymbol{\mu}\|\|\bar{\boldsymbol{\omega}}\|\cos\bar{\varphi}$, where $\|\cdot\|$ denotes the length of the respective vector, and $\bar{\varphi}$ the angle between vector $\boldsymbol{\mu}$ and vector $\bar{\boldsymbol{\omega}}$. As demonstrated in chapter 1, $\|\boldsymbol{\mu}\|$, $\|\bar{\boldsymbol{\omega}}\|$ and β^A determine the effort intensity but do not influence the relative effort allocation. In contrast, $\bar{\varphi}$ measures effort distortion.
To illustrate that $\bar{\varphi}$ measures also the efficiency loss imposed by the application of $\bar{S}$ in the agent's incentive contract, suppose first that $\bar{\boldsymbol{\omega}} = \xi^{-1}\boldsymbol{\mu}$, $\xi > 0$, such that $\mathrm{E}[\bar{S}|\boldsymbol{e}] = \xi^{-1}\boldsymbol{\mu}^t\boldsymbol{e}$, i.e. the expected signal is congruent with the expected firm value. Then, the principal's expected profit on the basis of a congruent information system becomes $\hat{\Pi} = \boldsymbol{\mu}^t\boldsymbol{\mu}/4$. Accordingly, $\Delta\Pi \equiv \hat{\Pi} - \Pi^*$ is the expected loss if $\bar{S}$ is not perfectly congruent. Consequently,

$$\Delta\Pi = \frac{1}{4}\left[\boldsymbol{\mu}^t\boldsymbol{\mu} - \frac{(\boldsymbol{\mu}^t\bar{\boldsymbol{\omega}})^2}{\bar{\boldsymbol{\omega}}^t\bar{\boldsymbol{\omega}}}\right], \tag{2.50}$$

which is equivalent to

$$\Delta\Pi = \frac{(\sum_i \mu_i^2)(\sum_i \bar{\omega}_i^2) - (\sum_i \mu_i\bar{\omega}_i)^2}{4\sum_i \bar{\omega}_i^2}. \tag{2.51}$$

According to the Cauchy-Schwarz inequality, $\Delta\Pi \geq 0$. Applying the relations $\sqrt{\sum_i \mu_i^2} = \|\boldsymbol{\mu}\|$ and $\sum_i \mu_i\bar{\omega}_i = \|\boldsymbol{\mu}\|\|\bar{\boldsymbol{\omega}}\|\cos\bar{\varphi}$ gives

$$\Delta\Pi = \frac{1}{4}\|\boldsymbol{\mu}\|^2(1 - \cos^2\bar{\varphi}) \geq 0, \tag{2.52}$$

which can be re-arranged to $\cos\bar{\varphi} \leq 1$. Consequently, $\bar{\varphi}$ measures $\Delta\Pi$. Finally, $\bar{\varphi} \in [0, \pi/2]$ since $\mu_i, \bar{\omega}_i \geq 0$, $i = 1, ..., n$.

Q.E.D.

Proof of Proposition 2.4.
To identify the effect of $\bar{\varphi}$ on m^*, define

$$F \equiv \frac{1}{4}\|\boldsymbol{\mu}\|^2\sin\left(2\varphi\left(m^*, \bar{\varphi}\right)\right)\left(-\varphi_m\right) - C'(m^*) = 0. \tag{2.53}$$

Applying the Implicit Function Theorem gives $dm^*/d\bar{\varphi} = -(\partial F/\partial\bar{\varphi})/(\partial F/\partial m)$. Since the second-order condition for m^* is satisfied, it follows $\partial F/\partial m < 0$. Moreover,

$$\frac{\partial F}{\partial\bar{\varphi}} = \frac{1}{4}\|\boldsymbol{\mu}\|^2\left[2\cos(2\varphi(\cdot))(-\varphi_m)\varphi_{\bar{\varphi}} + \sin(2\varphi(\cdot))(-\varphi_{m\bar{\varphi}})\right], \tag{2.54}$$

which is strictly positive for all $\varphi(\cdot) \in (0, \pi/4)$. Consequently, $dm^*/d\bar{\varphi} > 0$. To identify the effect of $\bar{\varphi}$ on $\Pi^C(\cdot)$, one can apply the Envelope Theorem, which gives

$$\frac{d\Pi^C(m^*(\bar{\varphi}), \bar{\varphi})}{d\bar{\varphi}} = -\frac{1}{4}\|\boldsymbol{\mu}\|^2\sin\left(2\varphi(\cdot)\right)\varphi_{\bar{\varphi}}. \tag{2.55}$$

Observe that $d\Pi^C(\cdot)/d\bar{\varphi} < 0$ for all $\varphi(\cdot) \in (0, \pi/4)$.

Q.E.D.

Proof of Proposition 2.5.
First observe that (2.41) is equivalent to

$$\beta^S(m^S, \eta) = \frac{\eta C'(m^S)}{\rho'(m^S)}, \tag{2.56}$$

with $\beta^S(\cdot)$ as the required bonus to induce an arbitrary m^S. Consequently, the expected bonus $B(m^S, \eta) = \beta^S(m^S, \eta)\rho(m^S)$ becomes

$$B(m^S, \eta) = \frac{\eta C'(m^S)\rho(m^S)}{\rho'(m^S)}. \tag{2.57}$$

Note that $\beta^S(\cdot)$ needs to be strictly positive in order to induce $m^S > 0$. Consequently, (2.42) is satisfied as long as (2.43) holds and therefore omits. The Lagrangian therefore becomes

$$\begin{aligned} \mathcal{L}(\alpha^S, m^S) &= \frac{1}{4}\|\boldsymbol{\mu}\|^2 \cos^2 \varphi\left(m^S, \bar{\varphi}\right) - \alpha^S - B(m^S, \eta) \\ &\quad + \lambda\left[\alpha^S + B(m^S, \eta) - \eta C(m^S)\right] + \xi\alpha^S. \end{aligned} \tag{2.58}$$

The first-order conditions with respect to α^S and m^S are

$$-1 + \lambda + \xi = 0, \tag{2.59}$$

$$\frac{1}{4}\|\boldsymbol{\mu}\|^2 \sin\left(2\varphi\left(m^S, \bar{\varphi}\right)\right)(-\varphi_m) - \frac{\partial B(\cdot)}{\partial m^S} + \lambda\left[\frac{\partial B(\cdot)}{\partial m^S} - \eta C'(m^S)\right] = 0, \tag{2.60}$$

and the complementary slackness conditions,

$$\lambda\left[\alpha^S + B(\cdot) - \eta C(m^S)\right] = 0, \tag{2.61}$$

$$\xi\alpha = 0. \tag{2.62}$$

To find a solution of this problem, suppose for a moment that $\lambda > 0$. Note that in this case, $\alpha^S + B(\cdot) - \eta C(m^S) = 0$ due to (2.61). Since it is required that $\alpha^S \geq 0$, this would imply that $B(\cdot) \leq \eta C(m^S)$. In this case, the supervisor maximizes her expected utility by choosing $m^S = 0$. Hence, $\lambda > 0$ cannot be a solution of this problem. Consequently, $\lambda = 0$, i.e. the supervisor's participation constraint is not binding. We can further infer from (2.59) that $\xi = 1$. Thus, in accordance to (2.62), the principal sets $\alpha^{S*} = 0$. Since $\lambda = 0$, we can observe from (2.60) that m^{S*} solves

$$\frac{1}{4}\|\boldsymbol{\mu}\|^2 \sin\left(2\varphi\left(m^{S*}, \bar{\varphi}\right)\right)(-\varphi_m) = \frac{\partial B(m^{S*}, \eta)}{\partial m^S}. \tag{2.63}$$

Next, it is necessary to proof that the first-order approach is also sufficient. Recall from assumption 2.2 that the gross payoff is concave increasing in m^S. Consequently, it is sufficient to show that $B(\cdot)$ is convex increasing in m^S. The first derivative of $B(\cdot)$ with respect to m^S gives

$$\frac{\partial B(\cdot)}{\partial m^S} = \eta C'(\cdot) + \frac{\rho(m^S)\eta C''(m^S)}{\rho'(m^S)} + \frac{\rho(m^S)\eta C'(m^S)(-\rho''(m^S))}{[\rho'(m^S)]^2}, \quad (2.64)$$

which is strictly positive for all $m^S > 0$, i.e. $B(\cdot)$ is increasing in m^S. The second derivative leads to

$$\begin{aligned}\frac{\partial^2 B(\cdot)}{\partial (m^S)^2} &= \eta C''(\cdot) + \frac{[\rho'(\cdot)\eta C''(\cdot) + \rho(\cdot)\eta C'''(\cdot)]\,\rho'(\cdot) - \rho(\cdot)\eta C''(\cdot)\rho''(\cdot)}{[\rho'(\cdot)]^2} \\ &\quad + \frac{[(\rho'(\cdot)\eta C'(\cdot) + \rho(\cdot)\eta C''(\cdot))\,(-\rho''(\cdot))]\,[\rho'(\cdot)]^2}{[\rho'(\cdot)]^4} \\ &\quad - \frac{2\rho(\cdot)\eta C'(\cdot)(-\rho''(\cdot))\rho'(\cdot)\rho''(\cdot)}{[\rho'(\cdot)]^4}, \end{aligned} \quad (2.65)$$

which can be re-arranged to

$$\begin{aligned}\frac{\partial^2 B(\cdot)}{\partial (m^S)^2} &= 2\eta C''(\cdot) + \frac{\rho(\cdot)\eta C'''(\cdot)}{\rho'(\cdot)} + \frac{\rho(\cdot)\eta C''(\cdot)(-\rho''(\cdot))}{[\rho'(\cdot)]^2} \\ &\quad + \frac{[\rho'(\cdot)\eta C'(\cdot) + \rho(\cdot)\eta C''(\cdot)]\,[-\rho''(\cdot)]}{[\rho'(\cdot)]^2} + \frac{2\rho(\cdot)\eta C'(\cdot)(-\rho''(\cdot))^2}{[\rho'(\cdot)]^3}. \end{aligned} \quad (2.66)$$

Observe that $\partial^2 B(\cdot)/\partial(m^S)^2 > 0$. Accordingly, $B(\cdot)$ is convex increasing in m^S. This implies that the first-order approach to identify the optimal measurement intensity is also sufficient. Finally, substituting $\alpha^{S*} = 0$ and m^{S*} in the principal's objective function gives $\Pi^D(m^{S*}, \bar{\varphi}, \eta)$.

Q.E.D.

Proof of Proposition 2.7.
For identifying $dm^{S*}/d\bar{\varphi}$, define

$$F \equiv \frac{1}{4}\|\boldsymbol{\mu}\|^2 \sin\left(2\varphi\left(m^*, \bar{\varphi}\right)\right)(-\varphi_m) - \frac{\partial B(\cdot)}{\partial m^S} = 0. \quad (2.67)$$

The Implicit Function Theorem gives $dm^{S*}/d\bar{\varphi} = -(\partial F/\partial\bar{\varphi})/(\partial F/\partial m^S)$. First, one get

$$\frac{\partial F}{\partial \bar{\varphi}} = \frac{1}{4}\|\boldsymbol{\mu}\|^2\left[2\cos(2\varphi(\cdot))(-\varphi_m)\varphi_{\bar{\varphi}} + \sin(2\varphi(\cdot))(-\varphi_{m\bar{\varphi}})\right], \quad (2.68)$$

which is strictly positive for all $m^S > 0$ and $\bar{\varphi} \in [0, \pi/2)$. Since the second-order condition for m^{S*} is satisfied, we have $\partial F/\partial m^S < 0$. Consequently, $dm^{S*}/d\bar{\varphi} > 0$. The effect of η on m^{S*} can be shown by applying $dm^{S*}/d\eta = -(\partial F/\partial\eta)/(\partial F/\partial m^S)$. Recall that $\partial F/\partial m^S < 0$. Furthermore, the first derivative of F with respect to η yields

$$\frac{\partial F}{\partial \eta} = -C'(m^S) - \frac{\rho(m^S)C''(m^S)}{\rho'(m^S)} - \frac{\rho(m^S)C'(m^S)(-\rho''(m^S))}{[\rho'(m^S)]^2}, \quad (2.69)$$

which is strictly negative for all $m^S > 0$. Consequently, $dm^{S*}/d\eta < 0$. To illustrate the effect of η on $\Pi^D(\cdot)$, one can apply the Envelope Theorem, which gives

$$\frac{d\Pi^D(\cdot)}{d\eta} = -\frac{\partial B(\cdot)}{\partial \eta} = -\frac{C'(m^S)\rho(m^S)}{\rho'(m^S)}. \tag{2.70}$$

Thus, $d\Pi^D(\cdot)/d\eta < 0$ for all $m^{S*} > 0$.

Q.E.D.

Comparison of Measurement Intensities.
First, recall that

$$\frac{\partial B(\cdot)}{\partial m^S} \;=\; \eta C'(\cdot) + \frac{\rho(m^S)\eta C''(m^S)}{\rho'(m^S)} + \frac{\rho(m^S)\eta C'(m^S)(-\rho''(m^S))}{[\rho'(m^S)]^2}. \tag{2.71}$$

Consequently, $C'(m^*(\cdot)) > \partial B(m^*(\cdot), \eta)/\partial m^S$ is equivalent to

$$\frac{1-\eta}{\eta} C'(m^*(\cdot)) > \frac{\rho(m^*(\cdot))C''(m^*(\cdot))}{\rho'(m^*(\cdot))} + \frac{\rho(m^*(\cdot))C'(m^*(\cdot))\left[-\rho''(m^*(\cdot))\right]}{[\rho'(m^*(\cdot))]^2}. \tag{2.72}$$

This can be transformed to

$$\frac{1-\eta}{\eta} > \frac{C''(m^*(\cdot))}{C'(m^*(\cdot))}\frac{\rho(m^*(\cdot))}{\rho'(m^*(\cdot))} + \frac{\rho(m^*(\cdot))\left[-\rho''(m^*(\cdot))\right]}{[\rho'(m^*(\cdot))]^2}, \tag{2.73}$$

and hence,

$$\frac{1-\eta}{\eta} > \frac{\rho(m^*(\cdot))}{\rho'(m^*(\cdot))}\left[\frac{C''(m^*(\cdot))}{C'(m^*(\cdot))} - \frac{\rho''(m^*(\cdot))}{\rho'(m^*(\cdot))}\right]. \tag{2.74}$$

Q.E.D.

Proof of Proposition 2.8.
Observe that η^* implies $F \equiv \Pi^D(m^{S*}(\bar{\varphi}, \eta^*), \bar{\varphi}, \eta) - \Pi^C(m^*(\bar{\varphi}), \bar{\varphi}) = 0$. Suppose first that $\eta \geq 1$. Due to rent extraction, inducing an arbitrary measurement intensity is more costly under delegation than under centralization. Accordingly, we have $B(m) > \eta C(m) \geq C(m)$ for all $m > 0$ and $\eta \geq 1$. In contrast, if $\eta = 0$, the supervisor can be induced to implement $m^S \to \infty$ with $B(m^S, 0) = 0$. In this case, delegation is strictly superior. Moreover, recall from proposition 2.7 that $d\Pi^D(\cdot)/d\eta < 0$ for all $m^{S*} > 0$. Consequently, there exists a cut off $\eta^* \in (0, 1)$ implying that the principal is indifferent between centralization and delegation.
To identify the effect of $\bar{\varphi}$ on η^*, one can apply the Implicit Function Theorem, which gives $d\eta^*/d\bar{\varphi} = -(\partial F/\partial\bar{\varphi})/(\partial F/\partial\eta)$. In order to derive $\partial F/\partial\bar{\varphi}$, it useful to apply the Envelope Theorem for $\Pi^C(\cdot)$ and $\Pi^D(\cdot)$ separately. Thus,

$$\frac{\partial \Pi^D(\cdot)}{\partial \bar{\varphi}} = -\frac{1}{4}\|\boldsymbol{\mu}\|^2 \sin\left(2\varphi\left(m^{S*}(\cdot), \bar{\varphi}\right)\right) \varphi_{\bar{\varphi}}(m^{S*}), \tag{2.75}$$

and

$$\frac{\partial \Pi^C(\cdot)}{\partial \bar{\varphi}} = -\frac{1}{4}\|\boldsymbol{\mu}\|^2 \sin\left(2\varphi\left(m^*(\cdot), \bar{\varphi}\right)\right) \varphi_{\bar{\varphi}}(m^*). \tag{2.76}$$

Since

$$\frac{\partial F}{\partial \bar{\varphi}} = \frac{\partial \Pi^D(\cdot)}{\partial \bar{\varphi}} - \frac{\partial \Pi^C(\cdot)}{\partial \bar{\varphi}}, \tag{2.77}$$

we have

$$\begin{aligned} \frac{\partial F}{\partial \bar{\varphi}} &= \frac{\|\boldsymbol{\mu}\|^2}{4} \Big[\sin\left(2\varphi\left(m^*(\cdot), \cdot\right)\right) \varphi_{\bar{\varphi}}\big|_{m^*(\cdot)} \\ &\qquad - \sin\left(2\varphi\left(m^{S*}(\cdot), \cdot\right)\right) \varphi_{\bar{\varphi}}\big|_{m^{S*}(\cdot)} \Big]. \end{aligned} \tag{2.78}$$

Observe that $\partial F/\partial\bar{\varphi} > 0$ if

$$\frac{\sin\left(2\varphi\left(m^*(\cdot), \bar{\varphi}\right)\right)}{\sin\left(2\varphi\left(m^{S*}(\cdot), \bar{\varphi}\right)\right)} > \frac{\varphi_{\bar{\varphi}}(m^{S*}(\cdot))}{\varphi_{\bar{\varphi}}(m^*(\cdot))}. \tag{2.79}$$

Recall first that $\varphi(m^{S*}(\cdot), \bar{\varphi}) < \varphi(m^*(\cdot), \bar{\varphi})$ if $m^{S*}(\cdot) > m^*(\cdot)$. Consequently,

$$\sin(2\varphi(m^{S*}(\cdot), \bar{\varphi})) < \sin(2\varphi(m^*(\cdot), \bar{\varphi})), \tag{2.80}$$

for $\varphi(\cdot) \in [0, \pi/2]$. Moreover, $\varphi_{\bar{\varphi}}\big|_{m^*(\cdot)} > \varphi_{\bar{\varphi}}\big|_{m^{S*}(\cdot)}$. This eventually implies that (2.79) is satisfied for $m^{S*}(\cdot) > m^*(\cdot)$. Therefore, $\partial F/\partial\bar{\varphi} > 0$ if $m^{S*}(\cdot) > m^*(\cdot)$. In contrast, $\partial F/\partial\bar{\varphi} \leq 0$ if $m^{S*}(\cdot) \leq m^*(\cdot)$. Finally observe that $\Pi^C(\cdot)$ is independent of η. Thus, the Envelope Theorem gives

$$\frac{\partial F}{\partial \eta} = \frac{\partial \Pi^D(\cdot)}{\partial \eta} = -\frac{\partial B(\cdot)}{\partial \eta} = -\frac{\rho(\cdot)C'(\cdot)}{\rho'(\cdot)}, \tag{2.81}$$

which is strictly negative for all $m^{S*}(\cdot) > 0$. Thus, $d\eta^*/d\bar{\varphi} > 0$ if $m^{S*}(\cdot) > m^*(\cdot)$; and $d\eta^*/d\bar{\varphi} \leq 0$, otherwise.

Q.E.D.

Essay 3

Vertical Collusion, Incongruent Preferences in Inter-Firm Trade, and the Theory of the Firm

3.1 Introduction

Since the seminal work of Coase [1937], a stream of economic literature has emerged to focus on explanations of why firms exist and utilize non-market in place of market transactions. The question is still valid today: What are the conditions driving firms to assimilate outside transactions even though integrated transactions impose further inefficiencies due to moral hazard or hidden characteristics of agents?[1] As Williamson [1979] argued, the main rationale for firms' decision on whether to utilize integrated or non-integrated transactions is rooted in the associated costs. According to Williamson [1979] it is beneficial to integrate transactions so long as an organization can achieve the same objective to lower costs than by using pure market transactions. Empirical evidence supports this argumentation by demonstrating that the characteristics of transaction costs are crucial for organizational design (see e.g. Joskow [1988]).

In addition to agency costs arising from moral hazard or the hidden characteristics of agents, Tirole [1986] illustrated a further source of internal inefficiencies that stems from agents' incentives for side-contracting–what is commonly referred to as collusion.[2] Even though Tirole [1986], among others, restricted his analysis

[1]See Eisenhardt [1989] for an early survey of agency literature with distinctions being made between these two streams. For surveys with the emphasis on moral hazard see e.g. Prendergast [1999], Lambert [2001], Gibbons [2005], and Christensen and Feltham [2005]; and for adverse selection refer e.g. to Salanié [1997] and Bolton and Dewatripont [2005] and the references therein.

[2]Particularly, collusion among agents within the same hierarchical level may serve one of the subsequent intentions: (*i*) to hide the efficiency characteristic of agents [Laffont and Meleu, 1997, Laffont and Martimort, 2000]; (*ii*) to share risk [Itoh, 1993]; and, (*iii*) to misreport production costs [Laffont and Martimort, 1997]. In contrast, vertical side-contracting usually occurs among a supervisor and an agent whereby the supervisor may observe specific information such as random productivity shocks [Tirole, 1986, Kofman and Lawarree, 1993, Villadsen, 1995], the true costs of an implemented project [Strausz, 1997], the agent's type [Faure-Grimaud et al., 2003], or the agent's effort implementation [Kessler, 2000]. Under such circumstances, the agent may have incentives

to the integrated case and determined conditions ensuring collusion-proofness, it is intuitive to expand on this preliminary foundation to concentrate on the next logical question of whether potential collusion leads to the application of market instead of integrated transactions. From this angle, collusive behavior within organizations constitutes inefficiencies and imposes costs for integrated transactions, which in turn would imply that the exploitation of the market may be a device for avoiding these expenditures.

Utilizing the market for bypassing internal inefficiencies—such as collusion—may have its own drawback. Spot market transactions between firms in competitive markets appear to be beneficial as long as exchanged goods are not required to be tailored to the needs of particular demanding firms. Things are different, however, when demanding firms require unique adjustments of exchanged goods necessary for their own processing. In this case, the value of spot market transactions is limited because supplying firms are motivated to adjust their products with the aim of maximizing the market price they can achieve, rather than satisfying the requirements of a single potential trading partner. Generally speaking, supplying firms' preferences with respect to configure their products are not necessarily congruent with the requisites of specific demanding firms. By utilizing inter-firm trade for acquiring goods, demanding firms are potentially compelled to purchase less suitable products than desired. Nevertheless, engaging in bilateral long-term relationships with one supplying firm may ensure the exchange of sufficiently tailored goods. This particularly requires relation-specific investments from the supplying firm aimed at customizing its goods. However, making relation-specific investments in order to satisfy one demanding firm's prerequisites is only beneficial if holdup is not anticipated. As shown by Baker et al. [2002], Halonen [2002] and Itoh and Morita [2005], vertical integration can be a remedy for holdup problems.[3]

The purpose of this essay is to illustrate the trade-off between internal and external inefficiencies by means of collusion among agents within firms and incongruent preferences between firms. It investigates the consequences associated with (*i*) firms' decisions on whether to integrate transactions or to utilize the market, and (*ii*) the properties of contractual arrangements. The analysis in this essay suggests that the effect of potential collusion is twofold. First, it may force firms to adopt market transactions even though integrated transactions would have been more efficient otherwise. Second, the contingency of collusion may also be beneficial under specific circumstances if it facilitates the achievement, or enhances the profitability, of superior relational contracts within and between firms. As further shown, incongruent preferences between firms for the properties of exchanged goods affect the efficiency of long-term market relations intended to motivate relation-specific investments. The investigation in this essay indicates that more congruent preferences with respect to the properties of exchanged goods, are disadvantageous from demanding firms' perspectives because they impose higher costs to motivate the supplier to make relation-specific investments. On the other hand, diverging requirements for

for side-contracting in order to motivate the supervisor to withhold or misrepresent her private information.

[3]The Fisher Body-General Motors case reported in Klein et al. [1978] is a widely used example for illustrating holdup. For a summary of extreme examples of holdup see Shavell [2005].

goods between demanding and supplying firms (incongruent preferences) do not affect the value of spot market transactions from the demanding firms' perspectives. This is observable because the exchange of goods with less suitable properties leads to a lower transfer price, thereby leaving the profitability of spot market transaction unaffected.

This essay contributes to contemporary literature in the theory of the firm in two ways. First, it defines how potential collusion within organizations can be a rationale for market transactions even though an internal production process appears to be more efficient. Second, it illuminates how incongruent preferences between firms for the characteristics of exchanged goods determine the contractual arrangements for their bilateral relationship.

This essay is related to following strands of literature. First, the framework in this essay builds on the model established by Baker et al. [2002], who also deal with optimal transactions within and between firms. They analyze how asset ownership facilitate the achievement of superior relational contracts, which in turn affects the properties of intra-firm and inter-firm transactions. The subsequently proposed framework therefore also comprises implicit (or relational) contracts within firms as analyzed e.g. by Bull [1987], Pearce and Stacchetti [1998], and Levin [2003]; and between firms as considered by Telser [1980] and Klein and Leffler [1981]. Second, this essay is related to the literature analyzing vertical side-contracting, as e.g. Tirole [1986], Kofman and Lawarree [1993] and Villadsen [1995]. Third, this essay also deals with relation-specific investments as analyzed by Edlin and Reichelstein [1996], Baker et al. [2001, 2002], Halonen [2002], and Itoh and Morita [2005]. Nevertheless, this essay differs from prior studies in two main aspects. First, it elaborates on collusion as a supplementary contribution to the theory of the firm. The objective is to illustrate how potential side-contracting within firms can be a rational for market transaction aimed at bypassing this internal inefficiency. Second, this essay exemplifies how different preferences affect the characteristics of inter-firm transactions.

This essay proceeds as follows. I introduce the basic model in section 3.2 and derive the first-best solution as a benchmark in section 3.3. The optimal contractual arrangements for the different types of transactions are derived in section 3.4, and compared in section 3.5 with the emphasis on how collusion within, and incongruent preferences between firms affect the properties of transactions. Furthermore, I illustrate in section 3.6 the value of verifiable but imperfect signals for the efficiency of transactions. Section 3.7 summarizes the main results and concludes.

3.2 The Model

Consider a risk-neutral market participant (firm) referred henceforth as the downstream party. In every period, she needs a preliminary product in order to sustain her production, e.g. engines or windshields for manufacturing cars. The downstream party owns an asset which can be utilized to produce this good. However, she lacks either the ability or the time to produce the good by herself.

The good is differently valued by the downstream party and other market participants, possibly due to diverse usage intentions. In particular, the downstream's (internal) good value I_i and the market's (external) value E_i can be either high ($i = H$) or low ($i = L$), where $\Delta I \equiv I_H - I_L$ and $\Delta E \equiv E_H - E_L$ are the respective spreads between the high and low value. Independent of the realized good value, the downstream party always prefers an internal use to maintain her production instead of selling it on the market. This requires that the low internal value exceeds the high external value, or formally, $I_H > I_L > E_H > E_L \geq 0$, where for parsimony, $E_L = 0$. The realized internal and external good value are observable by all involved entities but non-verifiable by third parties. This occurs for instance when the attainment of a certain quality standard is predominant in the valuation of a good. Quality is observable and involved parties are able to assess whether or not a previously defined quality standard is achieved. Nonetheless, it is sometimes either impossible to verify the achieved quality, or the associated costs are prohibitively high.

The downstream party can choose among two alternatives to obtain the required good: (i) she can purchase the good from another risk-neutral market participant (firm) referred henceforth as the upstream party (market transaction); or (ii), she can utilize an integrated production (employment). The upstream party owns a similar asset as the downstream party such that their production technologies are identical.

If the downstream party utilizes an integrated production, she requires the service of a worker as the productive entity. The worker is risk-neutral and financially constrained (limited liability). For parsimony, her reservation utility is zero.[4]

In every period, depending on the chosen transaction, either the worker (W) or the upstream party (U) implements non-verifiable effort $\boldsymbol{e}_i = (e_{i1}, ..., e_{in})^t$, $i = W, U$, $\boldsymbol{e}_i \in \mathbb{R}^{n+}$.[5] Effort imposes strictly convex increasing costs $C(\boldsymbol{e}_i) = \boldsymbol{e}_i^t \boldsymbol{e}_i/2$, $i = W, U$, which, for simplification purposes, are assumed to be identical for the worker and upstream party.[6] By choosing the respective effort intensity and allocation, the worker or upstream party determine the probability of whether the internal and external good value will be high or low. More precisely, let

$$\begin{aligned} \text{Prob}\{I_i = I_H | \boldsymbol{e}_i\} &= \min\{\boldsymbol{\mu}^t \boldsymbol{e}_i, 1\}, \\ \text{Prob}\{E_i = E_H | \boldsymbol{e}_i\} &= \min\{\boldsymbol{\omega}^t \boldsymbol{e}_i, 1\} \end{aligned}$$

be the probabilities that the high internal and external, and

$$\begin{aligned} \text{Prob}\{I_i = I_L | \boldsymbol{e}_i\} &= 1 - \min\{\boldsymbol{\mu}^t \boldsymbol{e}_i, 1\}, \\ \text{Prob}\{E_i = E_L | \boldsymbol{e}_i\} &= 1 - \min\{\boldsymbol{\omega}^t \boldsymbol{e}_i, 1\} \end{aligned}$$

be the probabilities that the low internal and external good value will be realized. These probabilities are further conditional independent, where $\boldsymbol{\mu} = (\mu_1, ..., \mu_n)^t$ and $\boldsymbol{\omega} = (\omega_1, ..., \omega_n)^t$, $\boldsymbol{\mu}, \boldsymbol{\omega} \in \mathbb{R}^{n+}$, characterize the marginal effect of $\boldsymbol{e}_i$ on the

[4] It is well known that financially constrained agents extract economic rents if their reservation utility is sufficiently low, see e.g. Demougin and Fluet [2001] and Demougin and Helm [2005]. Thus, assuming a small but strictly positive reservation utility does not change the subsequent results.

[5] All vectors are column vectors, where 't' denotes the transpose.

[6] The upstream's costs for implementing an arbitrary $\boldsymbol{e}_U$ can be either her own disutility of effort, or her costs for motivating a worker as productive party.

probabilities to realize the respective high good value. For parsimony, it is assumed that $\|\boldsymbol{\mu}\| = \|\boldsymbol{\omega}\| = 1$, i.e. the lengths of $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$ are normalized to one.

Next, consider an integrated production. Since the realized good value is non-verifiable, the downstream party cannot use this information to provide the worker with an explicit incentive contract. The downstream party, however, may recruit a supervisor who is charged with confirming the manifested good value (supervision). Due to her supposedly neutral position, the supervisor's confirmation eventually potentiates that the final internal good value is 'quasi-verifiable' from the courts' perspective such that it can be applied in explicit incentive contracts. The supervisor is risk-neutral and her best alternative provides her with the utility $\bar{U}^S$.

However, employing a supervisor for acquiring 'quasi-verifiable' information may have one drawback. Suppose the downstream party provides the worker with a bonus contract contingent on the realized internal good value affirmed by the supervisor. If the low value is realized, the worker might prefer to bribe the supervisor so that she reports a high good value. In this case, the downstream party is forced to pay the contracted bonus. If the final good is one of high value, the downstream party could be better off by bribing the supervisor into confirming a low good value in order to avoid the bonus payment. Consequently, empowering the supervisor to eventually confirm the realized good value may lead to side-contracting between the supervisor and either the worker or the downstream party, and contradicts the objective of supervision.

As pointed out earlier, one further alternative for the downstream party is to purchase the good from the upstream party. Utilizing a market relation with the upstream party, however, may imply one disadvantage: the upstream part is not necessarily intended to adapt her production with the aim of maximizing the downstream's internal good value. In fact, she wants to maximize the transfer price she may obtain in exchange for the good. The preferences between the downstream party and upstream party are therefore not necessarily aligned and deemed to be incongruent. This may motivate the upstream party to implement an inefficient effort allocation from the downstream's perspective. Suppose for a moment that there exists a constant $\lambda > 0$ satisfying $\boldsymbol{\mu} = \lambda\boldsymbol{\omega}$. In this case, maximizing the expected internal good value requires the same effort allocation as maximizing the expected external good value. This implies perfectly aligned preferences between the downstream and upstream party in terms of a particular effort allocation. If there exists no constant $\lambda > 0$ satisfying $\boldsymbol{\mu} = \lambda\boldsymbol{\omega}$, maximizing the internal and external good value are competing objectives in the sense that they require the implementation of a different effort allocation. Yet, the upstream's incentives depart from the preferences of the downstream party if she also wants to enhance the expected external good value rather than focussing exclusively on maximizing the expected internal value. The relation between vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\omega}$ in the n-dimensional space is characterized by the angle φ, which I refer henceforth as the dimension of incongruent preferences induced by the production technology.

The downstream party cannot only choose between an integrated and market transaction, she can also decide on the type of contract she applies. This can be either an explicit contract about verifiable information (spot contract), or a relational contract about observable, but non-verifiable information such as the realized

Transaction		Contract	
		Spot Contract	**Relational Contract**
Integration	Employment	*Spot Employment (SE)*	*Relational Employment (RE)*
	Supervision	*Supervision (S)*	
Market		*Spot Market (SM)*	*Relational Market (RM)*

Table 3.1: Combinations of Transactions and Contracts

internal good value I_i.[7] The subsequently considered combinations of transactions and contracts are summarized in table 3.1.

Relational (or implicit) contracts refer to contracts, for which some elements are not enforceable by third parties [MacLeod and Malcomson, 1989], or are prohibitively costly to specify *ex ante* [Baker et al., 2002]. As demonstrated by extant literature, relational contracts contingent on non-verifiable information can be self-enforcing, and therefore, credible in repeated games.[8] To incorporate relational contracts into the subsequent framework, I assume that all parties live forever such that they can interact for an infinite number of periods. Infinitely living parties, however, can be achieved by assuming overlapping generations of individuals who in turn live only a certain number of periods [Thomas and Worrall, 1988]. Suppose for a moment that all parties interact only for a finite number of periods and share the same belief about which period is the last. In the last period, no party stays on implicit agreements due to the lack of prospective punishments. Going one period back, we can observe the same situation: since there is no prospective punishment, honoring implicit agreements cannot be a dominant strategy. We can continue this procedure until we reach the first period. Accordingly, interacting for a finite time horizon implies that relational contracts are not feasible.[9]

Finally, all parties share the same interest rate r and play a trigger strategy: once they detect a violation of implicit obligations, they will never rely on implicit agreements with the violator again. Particulary, I follow Thomas and Worrall's [1988] argumentation that firms obtain a bad reputation on the labor market, once they breached contracts. Accordingly, the violation of implicit obligations becomes common knowledge on the labor market such that no other worker trusts the downstream party and refuses to enter into a relationship founded on implicit agreements. However, if no participant has incentives to deviate *ex post* from her previously stipulated obligations, relational contracts are self-enforcing and eliminate opportunistic behavior.

[7]See e.g. Hayes and Schaefer [2000] for empirical evidence that privately observed information are frequently components of incentive contracts.

[8]Refer e.g. to Telser [1980], Radner [1985], Bull [1987], Thomas and Worrall [1988], or MacLeod and Malcomson [1989] for comprehensive analyzes and discussions of relational contracts.

[9]Alternatively, some authors assume uncertainty about the duration of the interaction. This uncertainty can thereby also ensure that relational obligations are honored, see e.g. Telser [1980] and Schöttner [2005].

3.3 The First-Best Outcome

Before turning to the different organizational forms, let us briefly consider the first-best solution as a benchmark. Recall that the worker's and upstream's costs for implementing an arbitrary effort vector are identical. Therefore, the first-best solution is independent of whether the worker or the upstream party is in charge of producing the good. Since an internal use of the good is always optimal from the downstream's perspective, the first-best effort vector $\boldsymbol{e}^{fb}$ maximizes the difference between the expected internal good value and the corresponding production costs. Formally, $\boldsymbol{e}^{fb}$ solves

$$\max_{\boldsymbol{e}} \Pi \equiv I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^t\boldsymbol{e}. \tag{3.1}$$

Assume for a moment that $\text{Prob}\{I_i = I_H | \boldsymbol{e}^{fb}\} < 1$, i.e. e^{fb} characterizes an interior solution. Then, the downstream party assigns $\boldsymbol{e}^{fb} = \Delta I \boldsymbol{\mu}$. Observe that $\boldsymbol{e}^{fb}$ consists of two components: the scalar ΔI and the vector $\boldsymbol{\mu}$. The spread of the internal good value ΔI thereby determines the first-best effort intensity, whereas the relative effort allocation across tasks is characterized by $\boldsymbol{\mu}$. To see this, consider two arbitrary first-best activities e_i^{fb} and e_j^{fb}, $i \neq j$. The relative effort allocation is thereby characterized by

$$\frac{e_i^{fb}}{e_j^{fb}} = \frac{\mu_i}{\mu_j}, \quad i, j = 1, ..., n, \;\; i \neq j. \tag{3.2}$$

The relative effort allocation obviously shifts towards a higher focus on activity i, the higher its marginal effect μ_i on the probability to realize the high internal good value is, and vice versa. Roughly speaking, the first-best effort allocation reflects the relative importance of tasks with respect to producing the high internal good value. If, however, there exists no constant $\lambda > 0$ for any implemented effort vector $\boldsymbol{e}$ satisfying $\boldsymbol{e}^{fb} = \lambda \boldsymbol{e}$, the effort allocation deviates from first-best and is deemed to be distorted.

The first-best effort vector $\boldsymbol{e}^{fb}$ was previously assumed to characterize an interior solution. Note, however, that the implementation of $\boldsymbol{e}^{fb} = \Delta I \boldsymbol{\mu}$ leads to

$$\text{Prob}\{I_i = I_H | \boldsymbol{e}^{fb}\} = \min\{\Delta I, 1\} \tag{3.3}$$

As a consequence, we obtain a corner solution for $\Delta I \geq 1$. To ensure interior solutions, I assume henceforth that $\Delta I, \Delta E < 1$.[10]

Provided that $\Delta I < 1$ and the productive party implements $\boldsymbol{e}^{fb}$, the downstream's receives

$$\Pi^{fb} = I_L + \frac{1}{2}(\Delta I)^2, \tag{3.4}$$

which is convex increasing in the spread of the internal good value ΔI, and independent from the external good value E_i.

[10] Alternatively, one can let $\Delta I, \Delta E > 1$ by assuming that $\boldsymbol{\mu}^t\boldsymbol{\mu}$ and $\boldsymbol{\omega}^t\boldsymbol{\omega}$ are sufficiently small. In this case, the lengths of $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$ appear in the subsequent solutions. Since this does not provide additional insights, I decided for the first alternative for parsimony purposes.

3.4 Organizational Forms

In subsequent sections, I elaborate on the different types of transactions and contractual arrangements as summarized in table 3.1. The main focus is on the characteristics and efficiency of the contractual arrangements within and between firms.

3.4.1 Spot Employment

Consider first the case where the downstream party recruits a worker as the productive party. Since verifiable information about the realized good value are not available, the downstream party cannot provide the worker with an enforceable incentive contract. However, the downstream party can promise to pay a bonus if the high internal good value is realized. Once this occurs, the downstream party can take the good without paying the promised bonus since she owns the asset and possesses the related property rights [Grossman and Hart, 1986, Hart and Moore, 1990]. Anticipating this behavior, the worker is better off by implementing her least costly effort vector $\boldsymbol{e}_W = (0, ..., 0)^t$. Consequently, the downstream party obtains $\Pi^{D|SE} = I_L$ under spot employment.

3.4.2 Relational Employment

In contrast to spot employment, suppose the downstream party interacts with the worker for an infinite number of periods such that a relational contract might be self-enforcing. In order to provide appropriate incentives, the downstream party can promise the worker to pay a bonus β in addition to a fixed transfer α in the event that the high internal good value is realized. The worker's binary wage payment w^W takes thereby the form

$$w^W = \begin{cases} \alpha + \beta, & \text{if} \quad I_i = I_H \\ \alpha, & \text{if} \quad I_i = I_L, \end{cases} \tag{3.5}$$

where the worker's liability limit requires that all transfers have to be non-negative.

As a result of the implicit nature of this bonus contract, the downstream's promise to pay β needs to be reliable in order to be effective. A relational contract based on I_i can be credible, and therefore self-enforcing, if the downstream party stands to lose more by violating this agreement than by fulfilling her (non-enforceable) obligations. To derive a sufficient condition which guarantees the completion of the downstream's implicit obligations, suppose the worker initially trusts the downstream party that she will pay the promised bonus β if $I_i = I_H$. In this case, she is motivated to implement effort such that the probability for realizing the high internal good value is strictly positive. Consider now the case where a high internal good value is indeed realized. The downstream party only pays the promised bonus if she values a sustained relationship with the worker based on implicit obligations more than the short-term gain she obtains by reneging on β. The downstream party is therefore not tempted to renege on β if

$$-\beta + \frac{\Pi^{D|RE}}{r} \geq \frac{\tilde{\Pi}^{RE}}{r}, \tag{3.6}$$

where
$$\tilde{\Pi}^{RE} \equiv \max\{\Pi^{D|SE}, \Pi^{D|S}, \Pi^{D|SM}, \Pi^{D|RM}\}$$
is the downstream's expected profit she could obtain under her best alternative. After reneging occurred, no worker wants to enter into a relationship with the downstream party based on implicit contracts. Accordingly, the downstream party can henceforth either choose spot employment (SE) or may employ the supervisor (S) in order to receive contractible information. Alternatively, she can also utilize a market relation with the upstream party based on explicit (SM) or implicit (RM) contracts.

The left side of (3.6) represents the downstream's expected payoff when she acts honestly, i.e. paying the promised bonus β but obtaining $\Pi^{D|RE}$ in the future. In order to be deterred from reneging, this needs to be greater than the present value of her best fall-back position $\tilde{\Pi}^{RE}$. If this condition is satisfied, the worker anticipates that the downstream party delivers on her promise to pay β when $I_i = I_H$, and therefore, is motivated to implement effort. In contrast, if (3.6) is violated, the worker anticipates the downstream's reneging temptation and chooses her least costly effort vector $\boldsymbol{e}_W = (0, ..., 0)^t$. In this case, the downstream party receives $\Pi^{D|RE} = I_L$.

The optimal implicit bonus contract maximizes the difference between the expected internal good value and the worker's expected wage payment. However, the objective of the optimal relational employment contract is not only restricted to provide the worker with appropriate incentives, it is also required to be compatible with the previously derived self-enforcement condition. Consequently, the optimal bonus contract solves

$$\max_{\alpha,\beta,\boldsymbol{e}_W} \Pi^{D|RE} \equiv I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_W - \alpha - \beta \boldsymbol{\mu}^t \boldsymbol{e}_W \tag{3.7}$$

s.t.

$$\alpha + \beta \boldsymbol{\mu}^t \boldsymbol{e}_W - \frac{1}{2}\boldsymbol{e}_W^t \boldsymbol{e}_W \geq 0 \tag{3.8}$$
$$\boldsymbol{e}_W \in \arg\max_{\tilde{\boldsymbol{e}}_W} \alpha + \beta \boldsymbol{\mu}^t \tilde{\boldsymbol{e}}_W - \frac{1}{2}\tilde{\boldsymbol{e}}_W^t \tilde{\boldsymbol{e}}_W \tag{3.9}$$
$$\alpha + \beta \geq 0 \tag{3.10}$$
$$\alpha \geq 0 \tag{3.11}$$
$$I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_W - \alpha - \beta \boldsymbol{\mu}^t \boldsymbol{e}_W - \tilde{\Pi}^{RE} \geq \beta r. \tag{3.12}$$

Condition (3.8) is the worker's participation constraint and guarantees that it is in her interest to enter into this relationship. Furthermore, (3.9) is the worker's incentive constraint. Conditions (3.10) and (3.11) ensure that the bonus contract is compatible with the worker's liability limit. Finally, (3.12) is the self-enforcement condition and ensures that the relational bonus contract does not motivate the downstream party to renege *ex post* on β.

The worker's incentive constraint implies that she implements $\boldsymbol{e}_W = \beta \boldsymbol{\mu}$ in order to maximize her expected utility. The subsequent proposition further emphasizes the optimal relational bonus contract for all values of r.

Proposition 3.1 *Under relational employment, the optimal bonus contract is characterized by* $\alpha^* = 0$ *and*

$$\beta^*(r) = \begin{cases} \dfrac{\Delta I}{2}, & \text{if } \; r \leq r^{RE} \\ \dfrac{1}{2}(\Delta I - r) + \phi, & \text{if } \; r^{RE} < r \leq \widehat{r}^{RE} \\ 0, & \text{if } \; \widehat{r}^{RE} < r. \end{cases} \tag{3.13}$$

The downstream party receives

$$\Pi^{D|RE}(r) = \begin{cases} I_L + \dfrac{1}{4}(\Delta I)^2, & \text{if } \; r \leq r^{RE} \\ \dfrac{r}{2}\left[\Delta I - r + 2\phi\right] + \tilde{\Pi}^{RE}, & \text{if } \; r^{RE} < r \leq \widehat{r}^{RE} \\ I_L, & \text{if } \; \widehat{r}^{RE} < r, \end{cases} \tag{3.14}$$

where

$$r^{RE} \equiv \frac{\Delta I}{2} - \frac{2}{\Delta I}\left[\tilde{\Pi}^{RE} - I_L\right] \qquad \widehat{r}^{RE} \equiv \Delta I - 2\left[\tilde{\Pi}^{RE} - I_L\right]^{\frac{1}{2}}$$

$$\phi \equiv \left[\frac{1}{4}(\Delta I - r)^2 + I_L - \tilde{\Pi}^{RE}\right]^{\frac{1}{2}}.$$

Proof See appendix.

If $r \leq r^{RE}$, the downstream party sufficiently values a sustained relationship with the worker such that her promise to pay the optimal bonus $\beta^* = \Delta/2$ is credible. Anticipating that the downstream party adheres on her implicit obligations, the worker is motivated to implement $\boldsymbol{e}_W^* = \beta^* \boldsymbol{\mu}$. Note, however, that the downstream's expected profit for $r \leq r^{RM}$ is less than her expected first-best profit, see section 3.3. To identify the reason, observe that the optimal bonus contract provides the worker with the expected utility

$$EU^{W|RE} = \frac{1}{8}(\Delta I)^2. \tag{3.15}$$

Apparently, $EU^{W|RE} > 0$, i.e. the worker extracts a rent which is convex increasing in the spread of the internal good value ΔI. This is deducible from the fact that it is optimal from the downstream's perspective to provide more powerful incentives through a higher bonus β^* if ΔI increases. However, since negative payments are not compatible with the worker's liability limit, the downstream party must leave her a rent.

For $r^{RE} < r \leq \widehat{r}^{RE}$, however, the downstream party cannot credibly promise to pay $\beta^* = \Delta I/2$. To motivate the worker nevertheless to implement effort, she needs to adjust the bonus aimed at satisfying the self-enforcement condition (3.12). Observe that $\beta^*(r)$ is decreasing in r. A higher interest rate r implies a less severe 'punishment' for the downstream party for violating the implicit contract. To eliminate her *ex post* reneging temptation, the downstream party needs to adjust the bonus $\beta^*(r)$ appropriately. However, the more $\beta^*(r)$ deviates from the efficient bonus $\beta^* = \Delta I/2$, the lower is the downstream expected profit. If $r > \widehat{r}^{RE}$, the downstream party cannot find a strictly positive bonus which eliminates her reneging temptation. As a result, $\beta^* = 0$, and the downstream party obtains the spot employment profit.

3.4.3 Supervision

As shown, the downstream party cannot motivate the worker to implement effort if a relational contract contingent on I_i is not self-enforcing. Alternatively, the downstream party can employ a supervisor who is in charge of affirming the realized good value. This eventually ensures that the good value becomes 'quasi-verifiable' and can be applied in an enforceable incentive contract.

The supervisor is unproductive but can confirm the realized good value without costs. Verifying the worker's conducted effort, however, is prohibitively costly. If the supervisor is indifferent between a truthful and non-truthful confirmation, it is assumed that she reveals her observation truthfully. In exchange for her service, the downstream party offers the supervisor the payment w^S.

As a benchmark, let us first consider the collusion-free case.[11] After observing the realized good value, the supervisor reveals her observation truthfully such that the downstream party is forced to pay β^* if $I_i = I_H$. Accordingly, employing a supervisor is a commitment device for the downstream party to pay a promised bonus if the high internal good value is realized.

The downstream party wants to maximize the difference between the expected internal good value and the expected compensations for the worker and supervisor. Their optimal contracts thereby solve

$$\max_{\alpha,\beta,\boldsymbol{e}_W,w^S} \Pi^{D|S} \equiv I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_W - \alpha - \beta \boldsymbol{\mu}^t \boldsymbol{e}_W - w^S \tag{3.16}$$

s.t.

$$w^S \geq \bar{U}^S \tag{3.17}$$
$$(3.8), (3.9), (3.10), (3.11),$$

where (3.17) is the supervisor's participation constraint. Observe that this maximization problem is basically identical to the one considered in section 3.4.2 for relational employment, except that now w^S appears in the downstream's objective function, and the self-enforcement condition for relational employment does not apply. Additionally, it is required that $w^S \geq \bar{U}^S$ in order to ensure the supervisor's participation. To minimize her costs, the downstream party sets w^S such that the supervisor's participation constraint becomes binding, i.e. $w^S = \bar{U}^S$. Since w^S does not influence the worker's incentive contract, we obtain the same optimal bonus contract for the worker as under relational employment for $r \leq r^{RE}$. Accordingly,

[11] Throughout this essay it is important to distinguish between the collusion-free case and a collusion-proof contract. In the collusion-free case, side-contracting is excluded by assumption, whereas a collusion-proof contract refers to a contract ensuring that no party can improve her payoff by side-contracting [Tirole, 1986].

$\alpha^* = 0$ and $\beta^* = \Delta I/2$. This also leads to the implication that the worker receives the same expected utility. Given the optimal contracts for the worker and supervisor, the downstream party receives under collusion-free supervision

$$\Pi^{D|S} = I_L + \frac{1}{4}(\Delta I)^2 - \bar{U}^S. \tag{3.18}$$

Notice that employing a supervisor is only beneficial if her contribution to the downstream's expected profit exceeds her compensation $w^S = \bar{U}^S$. Formally, her employment is profitable in comparison to spot employment if

$$w^S = \bar{U}^S < \frac{1}{4}(\Delta I)^2. \tag{3.19}$$

Otherwise, the downstream party is at least better off by choosing spot employment. Since one purpose of this essay is to illustrate the consequences of collusion with the supervisor on the optimal transactions, I shall assume for the subsequent analysis that (3.19) holds.

Next, consider the case where collusion among the supervisor and either the worker or downstream party is potentially a dominant strategy. The realized internal good value eventually determines who might be better off by bribing the supervisor for spuriously claiming that the other value is realized. Particularly, if the low value is realized, the worker may have incentives to bribe the supervisor with the purpose of obtaining her bonus. In contrast, the downstream party may prefer to bribe the supervisor into confirming a low realized good value if the high value is realized. In this case, the downstream's objective is to avoid the payment of β^*. If the supervisor accepts the respective bribe, she does not deviate from the stipulated behavior and confirms the requested good value.[12] From the perspective of the external observers such as the courts, the worker and the downstream party are the immediate parties involved in the dispute over the value of the realized good whereas the supervisor is the supposedly neutral entity in this conflict. Due to individuals' tendency for personal gain, the court naturally assumes that the worker will always report a high value whereas the downstream party will always report the opposite. Therefore, the statement from the supervisor as a supposed 'neutral' party will have a greater weightage in swaying the court's decision. Collusion will thus work in favor of the colluding parties because the deceived party will have to convince the court of sufficient grounds in overturning her original judgment. To do so, however, the deceived party will not only have to prove the good's value, which itself is either exceedingly difficult or prohibitively expensive, but she will also have to demonstrate the existence of collusion, which is virtually impossible.

All involved parties can perfectly infer that collusion occurred once the confirmed good value deviates from the observed one. The occurrence of collusion is thereby considered as a breach of the implicit promise not to engage in side-contracting. Due

[12]It is not unusual in the collusion literature to assume that collusion is *per se* enforceable, see Tirole [1992] for a discussion, and for exemplary models of enforceable side-contracting see e.g. Tirole [1986], Kofman and Lawarree [1993], Villadsen [1995], Faure-Grimaud et al. [2001] and Faure-Grimaud et al. [2003]. There exists also experimental evidence that promises are honored among agents, see Dawes and Thaler [1988] for a survey. Nevertheless, some authors explicitly focus on enforcement mechanisms of side-contracting as e.g. Tirole [1992] and Martimort [1999].

to the trigger-strategy of all involved parties, the violators—the one who eventually bribed the supervisor successfully and the supervisor herself—are never trusted to honor implicit agreements again. Suppose the worker is the one who engaged in side-contracting with the supervisor. The downstream party dismisses both of them and, if she still prefers supervision, employs a different worker and supervisor from the labor market. However, things are different when the downstream party colludes with the supervisor. In this case, no worker in the labor market relies on implicit agreements with the downstream party again. Consequently, her alternatives shrink to spot employment as internal transaction, and either spot or relational transactions with the upstream party.

Next, let us derive collusion-proofness conditions preventing side-contracting among the supervisor and either the worker or the downstream party. Let T^i be the bribe the worker ($i = W$) or the downstream party ($i = D$) offers the supervisor in exchange for affirming the requested good value. For the subsequent analysis, assume the worker and the downstream party possess the entire bargaining power such that the supervisor can only accept or reject their respective offer.[13] The supervisor refuses T^i if

$$T^i + \frac{\bar{U}^S}{r} \leq \frac{w^S}{r}, \tag{3.20}$$

i.e. the transfer T^i does not compensate the supervisor for her prospective loss after she is dismissed and obtains $\bar{U}^S$ henceforth.

Suppose now that the low internal good value is realized such that it potentially provides the worker incentives to collude with the supervisor. The maximum bribe $\bar{T}^W$ she is willing to pay implies indifference between colluding and non-colluding. Formally, $\bar{T}^W$ satisfies

$$-\bar{T}^W + \beta = \frac{EU^W}{r}. \tag{3.21}$$

The maximum bribe $\bar{T}^W$ is equal to her short-term gain β and her discounted loss of expected utility after collusion is detected by the downstream party and she is dismissed. Observe that $\bar{T}^W \leq 0$ if $r \leq EU^W/\beta$. In this case, the worker sufficiently values a sustained employment such that she has no incentives to engage in side-contracting.

Now consider the downstream's temptation to collude with the supervisor. Suppose the worker had no incentives to collude with the supervisor in the first place and trusted the downstream party that she does not engage in side-contracting either. Nevertheless, it might be beneficial for the downstream party to pay the supervisor a bribe T^D aimed at avoiding the payment of β^* if the high internal good value is realized. The maximum bribe $\bar{T}^D$ the downstream party is willing to pay implies that she is indifferent between colluding and non-colluding. Formally, $\bar{T}^D$ satisfies

$$-\bar{T}^D + \frac{\tilde{\Pi}^S}{r} = -\beta + \frac{\Pi^{D|S}}{r}, \tag{3.22}$$

[13] For collusion models where the supervisor holds the entire bargaining power, refer to Faure-Grimaud et al. [2001] and Faure-Grimaud et al. [2003]. In contrast, Kofman and Lawarree [1996] apply the Nash bargaining solution to model different allocations of bargaining power.

where $\tilde{\Pi}^S$ denotes the downstream's expected profit she would receive under her best alternative. If $\bar{T}^D > 0$, the worker anticipates collusion between the downstream party and the supervisor. Hence, she is better off by refusing to implement effort such that $\text{Prob}[I_i = I_H | \boldsymbol{e}_W = \mathbf{0}] = 0$.

Proposition 3.2 *The worker's optimal contract* (α^*, β^*) *is collusion-proof if* $r \leq r^S$, *where*

$$r^S \equiv \min\{r^W, r^D\}, \qquad r^W \equiv \frac{\Delta I}{4}, \qquad r^D \equiv \frac{\Delta I}{2} - \frac{2}{\Delta I}\left[\tilde{\Pi}^S + \bar{U}^S - I_L\right].$$

If $r^S < r \leq \widehat{r}^S$, *the collusion-proof contract is characterized by* $\alpha^* = 0$ *and*

$$\beta^*(r) = \begin{cases} 2r, & \textit{if } \ r^W < r^D \\ \eta + \phi, & \textit{if } \ r^W \geq r^D. \end{cases} \tag{3.23}$$

Then, the downstream party receives

$$\Pi^{D|S}(r) = \begin{cases} I_L + 2r(\Delta I - 2r) - \bar{U}^S, & \textit{if } \ r^W < r^D \\ \eta + \phi + \tilde{\Pi}^S, & \textit{if } \ r^W \geq r^D, \end{cases} \tag{3.24}$$

where

$$\widehat{r}^S \equiv \frac{1}{2}\left[\frac{\Delta I}{3} - \left[\frac{1}{3^2}(\Delta I)^2 + \frac{2}{3}\kappa\right]^{\frac{1}{2}}\right] \qquad \eta \equiv \frac{1}{2}(\Delta I - r)$$

$$\kappa \equiv I_L - \bar{U}^S - \tilde{\Pi}^S \qquad \phi \equiv \left[\eta^2 + \kappa\right]^{\frac{1}{2}}.$$

If $r > \widehat{r}^S$, *the downstream party sets* $\beta^* = 0$ *and receives* $\Pi^{D|S} = I_L$. *Finally, the supervisor's contract is characterized by* $w^S = \bar{U}^S$ *for any value of* r.

Proof See appendix.

If $r \leq r^S$, all parties sufficiently value a sustained relationship such that no one is tempted to collude. In this case, the downstream party provides the worker with the same bonus contract as for collusion-free supervision, and therefore, the downstream party receives the same expected profit.

If in contrast $r^S < r \leq \widehat{r}^S$, the downstream party adjusts the worker's bonus aimed at preventing collusion. To discuss the two separated cases emphasized by the previous proposition, note that r^D refers to the downstream's temptation to collude, and r^W to the worker's, respectively. If in the first place $r^W < r^D$, i.e. the worker but not the downstream party is better off by side-contracting for $r^W < r \leq r^D$, the downstream party needs to enhance $\beta^*(r)$ in order to prevent the worker from side-contracting. This in turn improves her rent and makes side-contracting with the supervisor less beneficial. In contrast, the downstream party needs to reduce $\beta^*(r)$ in order to achieve collusion-proofness if $r^D \leq r^W$. This is necessary in order to prevent the downstream party herself from side-contracting. However, deviating from the efficient bonus $\beta^* = \Delta I/2$ in both directions inevitably leads to a lower expected profit the downstream party receives under supervision.

Note further that contemporaneously detaining the worker and downstream party to collude with the supervisor requires the adjustment of the bonus in opposite directions. As a consequence, the downstream party cannot find a bonus contract for $r > \widehat{r}^S$ which is collusion-proof. Yet, she can only avoid collusion by appointing $\beta^* = 0$, which will not only remove incentives to collude, but also to implement effort. In this case, the downstream party sets $w^S = 0$ since employing the supervisor in the first place is not beneficial from the downstream's perspective. Then, the downstream party obtains the same profit as under spot employment.

3.4.4 Spot Market

Instead of utilizing an integrated production, the downstream party can alternatively purchase the good from the upstream party. The subsequently considered transaction has the properties of a spot relationship in the sense that every period the downstream and upstream party negotiate about a transfer price Υ^{SM} in exchange for the good.

Consider first the consensual price agreed upon by the downstream and upstream party. To achieve this price, I apply the Nash-Bargaining solution with equal bargaining power. This suggests that gains from a coalition are shared equally among parties. In particular, the downstream party pays the upstream party the external good value E_i plus half of the downstream's surplus $I_i - E_i$ for its internal use. Consequently, $\Upsilon^{SM} = [I_i + E_i]/2$, and the expected transfer price takes the form

$$\mathrm{E}[\Upsilon^{SM}|\boldsymbol{e}_U] = \frac{1}{2}\left[I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_U + \Delta E \boldsymbol{\omega}^t \boldsymbol{e}_U\right]. \tag{3.25}$$

The higher the spread of the internal (ΔI) and external (ΔE) good value, the higher is the expected transfer price. There are two reasons for this observation. Firstly, a higher spread of the internal value makes the purchase of the good more beneficial for the downstream party, which is reflected by a higher expected transfer price. Secondly, a higher spread of the external value improves the upstream's bargaining position and hence, the price she expects to receive.

The upstream party chooses her effort vector $\boldsymbol{e}_U$ such that her expected profit for spot market transactions $\Pi^{U|SM} \equiv \mathrm{E}[\Upsilon^{SM}|\boldsymbol{e}_U] - C(\boldsymbol{e}_U)$ is maximized. Thus,

$$\max_{\boldsymbol{e}_U} \Pi^{U|SM} \equiv \frac{1}{2}\left[I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_U + \Delta E \boldsymbol{\omega}^t \boldsymbol{e}_U\right] - \frac{1}{2}\boldsymbol{e}_U^t \boldsymbol{e}_U, \tag{3.26}$$

which directly implies that the upstream party implements

$$\boldsymbol{e}_U^* = \frac{1}{2}\left[\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega}\right]. \tag{3.27}$$

Apparently, the upstream party implements the first-best effort vector $\boldsymbol{e}^{fb} = \Delta I \boldsymbol{\mu}$ if $\boldsymbol{\mu}\Delta I = \boldsymbol{\omega}\Delta E$. In this case, the upstream's effort allocation and effort intensity imply the same expected internal and external good value. If in contrast there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda\boldsymbol{\omega}$, the upstream party implements a suboptimal effort allocation; and if $\Delta I \neq \Delta E$, a suboptimal effort intensity.

If the upstream party anticipates a spot market transaction with the downstream party, she implements $\boldsymbol{e}_U^*$ and receives

$$\Pi^{U|SM} = \frac{1}{2}I_L + \frac{1}{8}\left[(\Delta I)^2 + (\Delta E)^2 + 2\Delta I \Delta E \cos\varphi\right]. \tag{3.28}$$

Recall that φ is the angle between vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\omega}$ and characterizes, how congruent the upstream's production technology is. It further measures, how efficient the upstream's motivated effort allocation from the downstream's perspective is. If $\varphi = 0$, the upstream party is motivated to implement the non-distorted (first-best) effort allocation, which is most preferred by the downstream party. Generally speaking, $\varphi = 0$ implies perfectly congruent preferences between both market participants. In contrast, the upstream party is motivated to implement an inefficient effort allocation from the downstream's perspective if $\varphi > 0$. This in turn provides her a lower expected profit because her production technology is less efficient in terms of contemporaneously maximizing the expected internal and external good value, and consequently, the expected transfer price.

If she decides for a spot market transaction, the downstream's expected profit is the difference between the expected internal good value and the transaction price she expects to pay in exchange for the good. Formally, $\Pi^{D|SM} \equiv \mathrm{E}[I_i - \Upsilon^{SM}|\boldsymbol{e}_U^*]$, which is equivalent to

$$\Pi^{D|SM} \equiv \frac{1}{2}I_L + \frac{1}{4}\left[(\Delta I)^2 - (\Delta E)^2\right]. \tag{3.29}$$

Finally observe that the downstream's expected profit is independent of the congruity measure φ. Nevertheless, there are two countervailing effects of φ on $\Pi^{D|SM}$. To see this, recall that $\mathrm{E}[I_i|\boldsymbol{e}_U^*] = I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_U^*$, and consequently,

$$\mathrm{E}[I_i|\boldsymbol{e}_U^*] = I_L + \frac{1}{2}(\Delta I)^2 + \frac{1}{2}\Delta I \Delta E \cos\varphi. \tag{3.30}$$

Likewise, the upstream's effort choice $\boldsymbol{e}_U^*$ leads to the expected transfer price

$$\mathrm{E}[\Upsilon^{SM}|\boldsymbol{e}_U^*] = \frac{1}{4}\left[(\Delta I)^2 + (\Delta E)^2\right] + \frac{1}{2}\left[I_L + \Delta I \Delta E \cos\varphi\right]. \tag{3.31}$$

Now suppose that φ increases, i.e. the upstream party is motivated to implement a more distorted effort allocation from the downstream's perspective. The marginal effect on the expected internal good value and expected transfer price therefore is

$$\frac{\partial\, \mathrm{E}[I_i|\boldsymbol{e}_U^*]}{\partial\varphi} = \frac{\partial\, \mathrm{E}[\Upsilon^{SM}|\boldsymbol{e}_U^*]}{\partial\varphi} = -\frac{1}{2}\Delta I \Delta E \sin\varphi. \tag{3.32}$$

Accordingly, an increase in φ leads firstly to a lower expected internal good value, but secondly, also to a lower expected transfer price. As we can infer from (3.30) and (3.31), both effects cancel each other such that $\Pi^{D|SM}$ does not change in φ. Roughly speaking, a lower expected transfer price perfectly compensates the downstream party for a lower expected good value.

3.4.5 Relational Market

As illustrated in the previous section, using a spot market transaction based on a negotiated transfer price may motivate the upstream party to implement an inefficient effort allocation from the perspective of the downstream party. As an alternative to negotiate every period about a transfer price, the downstream party can promise the upstream party to pay a certain amount exclusively contingent on the realized internal good value aimed at motivating the upstream party to implement the non-distorted (first-best) effort allocation. This bilateral relationship therefore requires relation-specific investments from the upstream party in the sense that she needs to focus exclusively on maximizing the expected internal good value, which leads to a deterioration of the expected market price she could obtain.

In particular, let P_L be a floor payment both parties consent to in an enforceable contract for exchanging the good, regardless of its final value. In addition, the downstream party promises to pay a higher price P_H if $I_i = I_H$. Thus, this type of transaction consists of an explicit component P_L and a non-enforceable premium $\Delta P \equiv P_H - P_L$. Notice that P_L is not necessarily strictly positive, but can also be negative in the sense of penalizing the upstream party for low internal good values. Provided that this relational contract is credible, it ensures the alignment of the downstream's and upstream's preferences for the properties of exchanged goods, and therefore, leads to the implementation of the first-best effort allocation.

The payments P_L and P_H need to guarantee that it is in the upstream's interest to enter into a sustained relationship with the downstream party based on a relational contract. This is the case, if the upstream party is at least weakly better off than under her best alternative. However, her best alternative is determined by the downstream's best fall-back position. If the downstream's next alternative to relational market is to utilize spot market transactions, the upstream's reservation utility is $\Pi^{U|SM}$ as derived in the previous section. In contrast, if an integrated transaction is the downstream's best alternative, the upstream's reservation utility is the one she receives for selling the good on the market at the price $\Upsilon^M = E_i$. In this case, the upstream party maximizes her expected profit $\Pi^{U|M} \equiv \mathrm{E}[\Upsilon^M|\boldsymbol{e}_U] - C(\boldsymbol{e}_U)$, which is equivalent to

$$\max_{\boldsymbol{e}_U} \Pi^{U|M} \equiv \Delta E \boldsymbol{\omega}^t \boldsymbol{e}_U - \frac{1}{2}\boldsymbol{e}_U^t \boldsymbol{e}_U. \tag{3.33}$$

If the upstream's objective is to sell the good on the market for Υ^M, she implements $\boldsymbol{e}_U^* = \Delta E \boldsymbol{\omega}$ and receives

$$\Pi^{U|M} = \frac{1}{2}(\Delta E)^2. \tag{3.34}$$

Suppose that the downstream party wants to engage in a sustained bilateral relationship with the upstream party and offers her the payments P_L and P_H. If the upstream party accepts, her intention to focus exclusively on maximizing the expected internal good value characterizes relation-specific investments and may lead to holdup: once the high internal good value is realized, the downstream party may prefer to renege on the promised premium ΔP. Nevertheless, the downstream party stays on her implicit obligation to pay ΔP if she values a sustained bilateral relationship more than she is tempted to renege. Suppose the high internal good value is realized. Then, the downstream party stays on her promise and pays ΔP if

$$-\Delta P + \frac{\Pi^{D|RM}}{r} \geq \frac{\tilde{\Pi}^{RM}}{r}, \tag{3.35}$$

where $\tilde{\Pi}^{RM}$ denotes the downstream's expected profit she receives under her best fall-back position. Note that $\Pi^{D|RM}$ can take one of two values conditional on the downstream's best alternative $\tilde{\Pi}^{RM}$. The downstream party adheres to her promise if paying the premium and maintaining the relationship provides her with a higher present expected profit than her best fall-back position.

As previously emphasized, the payments P_L and P_H need to provide the upstream party with appropriate incentives while ensuring her participation. Additionally, the proposed relational contract must be self-enforcing in the sense that the downstream party has no incentives to deviate from the stipulated behavior to pay the promised premium ΔP if $I_i = I_H$. Accordingly, the optimal payments solve

$$\max_{P_L, P_H, \boldsymbol{e}_U} \Pi^{D|RM} \equiv I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_U - P_L - \Delta P \boldsymbol{\mu}^t \boldsymbol{e}_U \tag{3.36}$$

s.t.

$$P_L + \Delta P \boldsymbol{\mu}^t \boldsymbol{e}_U - \frac{1}{2}\boldsymbol{e}_U^t \boldsymbol{e}_U \geq \bar{\Pi}^U \tag{3.37}$$

$$\boldsymbol{e}_U \in \arg\max_{\tilde{\boldsymbol{e}}_U} P_L + \Delta P \boldsymbol{\mu}^t \tilde{\boldsymbol{e}}_U - \frac{1}{2}\tilde{\boldsymbol{e}}_U^t \tilde{\boldsymbol{e}}_U \tag{3.38}$$

$$I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_U - P_L - \Delta P \boldsymbol{\mu}^t \boldsymbol{e}_U - \tilde{\Pi}^{RM} \geq \Delta P r, \tag{3.39}$$

where

$$\bar{\Pi}^U = \begin{cases} \frac{1}{2} I_L + \frac{1}{8}\left[\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega}\right]^2, & \text{if} \quad \tilde{\Pi}^{RM} = \Pi^{D|SM} \\ \frac{1}{2}(\Delta E)^2, & \text{if} \quad \tilde{\Pi}^{RM} \neq \Pi^{D|SM}, \end{cases} \tag{3.40}$$

is the upstream's reservation utility conditional on the downstream's best fall-back. Condition (3.37) is the upstream's participation constraint ensuring that the proposed relational contract makes her at least weakly better off than her best alternative. Moreover, (3.38) is her incentive condition. Finally, condition (3.39) ensures that the relational contract is self-enforcing.

The upstream's incentive constraint (3.38) indicates that she chooses $\boldsymbol{e}_U^* = \Delta P \boldsymbol{\mu}$ in order to maximize her expected profit. Apparently, this bilateral market relation is suitable to motivate the upstream party to implement non-distorted effort. As previously exposed, the upstream's reservation utility can take one of two values conditional on the downstream's best fall-back position. For the sake of lucidity, I subsequently consider both cases separately.

Proposition 3.3 *If* $\tilde{\Pi}^{RM} = \Pi^{D|SM}$*, the optimal premium* ΔP^* *under relational market is characterized by*

$$\Delta P^*(r) = \begin{cases} \Delta I, & \text{if} \quad r \leq r^{RM} \\ \Delta I - r + \phi, & \text{if} \quad r^{RM} < r \leq \hat{r}^{RM} \\ 0, & \text{if} \quad \hat{r}^{RM} < r. \end{cases} \tag{3.41}$$

Then, the downstream party receives

$$\Pi^{D|RM}(r) = \begin{cases} \frac{1}{2}\left[I_L + (\Delta I)^2\right] - \frac{1}{8}\left[\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega}\right]^2, & \text{if} \quad r \leq r^{RM} \\ r\left[\Delta I - r + \phi\right] + \Pi^{D|SM}, & \text{if} \quad r^{RM} < r \leq \widehat{r}^{RM} \\ \Pi^{D|SM}, & \text{if} \quad \widehat{r}^{RM} < r, \end{cases} \tag{3.42}$$

where

$$r^{RM} \equiv \frac{1}{8\Delta I}\left[\Delta I \boldsymbol{\mu} - \Delta E \boldsymbol{\omega}\right]^2 \qquad \widehat{r}^{RM} \equiv \Delta I - \eta^{\frac{1}{2}}$$

$$\phi \equiv \left[(\Delta I - r)^2 - \eta\right]^{\frac{1}{2}} \qquad \eta \equiv \frac{1}{4}\left(\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega}\right)^2 - I_L + 2\Pi^{D|SM}.$$

Proof See appendix.

If $r \leq r^{RM}$, the downstream party honors her non-enforceable obligation to pay $\Delta P^* = \Delta I$. In this case, the upstream party anticipates that holdup is not an issue and is motivated to make relation-specific investments. If in contrast $r^{RM} < r \leq \widehat{r}^{RM}$, the downstream party needs reduce the premium $\Delta P^*(r)$ in order to ensure that the relational contract remains credible, thereby implying a lower expected profit.[14] If $r > \widehat{r}^{RM}$, the downstream party cannot find a strictly positive and credible premium to provide the upstream party with incentives. Accordingly, $\Delta P^* = 0$. Since this would imply that the upstream party implements $\boldsymbol{e}_U^* = (0, ..., 0)^t$, the downstream party is better off by appointing $P_L^* = 0$ and to engage in spot market transactions with the upstream party. Consequently, $\Pi^{D|RM} = \Pi^{D|SM}$ for $r > \widehat{r}^{RM}$.

To gain further insights, we can re-write $\Pi^{D|RM}$ for $r \leq r^{RM}$ as[15]

$$\Pi^{D|RM} = \frac{1}{2}\left[I_L + (\Delta I)^2\right] - \frac{1}{8}\left[(\Delta I)^2 + (\Delta E)^2 + 2\Delta I \Delta E \cos\varphi\right]. \tag{3.43}$$

Apparently, $\Pi^{D|RM}$ depends on the congruity measure φ if the downstream's best fall-back is to utilize spot market transactions ($\tilde{\Pi}^{RM} = \Pi^{D|SM}$). The rationale for this observation is as follows: The downstream party has to provide the upstream party at least the same expected profit as she would receive if both engage in mutual spot market transactions. As a consequence, the floor payment P_L reflects the upstream's best alternative $\Pi^{U|SM}$, which in turn depends on φ, see section 3.4.4. Consequently, it also appears in $\Pi^{D|RM}$ if $\Pi^{D|SM} = \tilde{\Pi}^{RM}$. Furthermore, we have seen that a more congruent production technology (smaller φ) enhances the upstream's expected profit for spot market transactions. This in turn makes it more costly for the downstream party to ensure her participation, thereby implying a smaller expected profit $\Pi^{D|RM}$. The same observations apply for $\Pi^{D|RM}$ if $r^{RM} < r \leq \widehat{r}^{RM}$.

Finally, re-consider the cut off interest rate r^{RM}, which is equivalent to

$$r^{RM} = \frac{1}{8\Delta I}\left[(\Delta I)^2 + (\Delta E)^2 - 2\Delta I \Delta E \cos\varphi\right]. \tag{3.44}$$

[14] Note that changing ΔP requires also an adjustment of P_L to ensure the upstream's participation.

[15] To see this, recall that $\boldsymbol{\mu}^t \boldsymbol{\omega} = \|\boldsymbol{\mu}\|\|\boldsymbol{\omega}\| \cos\varphi$ and $\|\boldsymbol{\mu}\| = \|\boldsymbol{\omega}\| = 1$.

Observe that r^{RM} is increasing in φ. This can be observed because a higher φ—implied by a less congruent production technology—deteriorates the upstream's expected profit. This further implies that the downstream party can abate the floor payment P_L. As a consequence, the downstream party receives a higher expected profit such that her prospective 'punishment' for violating the implicit contract becomes more severe. This is in turn reflected by a higher r^{RM}. It can be verified that the same observation applies for $\widehat{r}^{RM}$.

Proposition 3.4 *If $\tilde{\Pi}^{RM} \neq \Pi^{D|SM}$, the optimal premium ΔP^* under relational market is characterized by*

$$\Delta P^*(r) = \begin{cases} \Delta I, & \textit{if} \;\; r \leq r^{RM} \\ \Delta I - r + \phi, & \textit{if} \;\; r^{RM} < r \leq \widehat{r}^{RM} \\ 0, & \textit{if} \;\; \widehat{r}^{RM} < r. \end{cases} \tag{3.45}$$

Then, the downstream party receives

$$\Pi^{D|RM}(r) = \begin{cases} I_L + \frac{1}{2}\left[(\Delta I)^2 - (\Delta E)^2\right], & \textit{if} \;\; r \leq r^{RM} \\ r\left[\Delta I - r + \phi\right] - (\Delta E)^2 + \tilde{\Pi}^{RM}, & \textit{if} \;\; r^{RM} < r \leq \widehat{r}^{RM} \\ \Pi^{D|SM}, & \textit{if} \;\; \widehat{r}^{RM} < r, \end{cases} \tag{3.46}$$

where

$$r^{RM} \equiv \frac{1}{2\Delta I}\left[2I_L + (\Delta I)^2 - (\Delta E)^2 - 2\tilde{\Pi}^{RM}\right] \qquad \widehat{r}^{RM} \equiv \Delta I - \eta^{\frac{1}{2}}$$

$$\phi \equiv \left[(\Delta I - r)^2 - \eta\right]^{\frac{1}{2}} \qquad \eta \equiv \left[(\Delta E)^2 + 2\tilde{\Pi}^{RM} - 2I_L\right].$$

Proof See appendix.

The downstream party sufficiently values a bilateral relationship with the upstream party based on implicit agreements if $r \leq r^{RM}$. As a consequence, the promise to pay the efficient premium $\Delta P^* = \Delta I$ is trustworthy and the upstream party implements $\boldsymbol{e}^* = \boldsymbol{\mu}\Delta I$. For $r^{RM} < r \leq \widehat{r}^{RM}$, however, the relational contract can be only self-enforcing if the downstream party adjusts $\Delta P^*(r)$, and therefore P_L, appropriately. If $r > \widehat{r}^{RM}$, the downstream party cannot credibly promise to pay a strictly positive premium. Thus, she sets $\Delta P^* = 0$ and engages in spot market transactions with the upstream party. Accordingly, the downstream party receives $\Pi^{D|RM} = \Pi^{D|SM}$ for $r > \widehat{r}^{RM}$.

3.5 The Optimal Organizational Form

After elaborating on the different combinations of transactions and contractual arrangements as summarize in table 3.1, we are better equipped to identify the efficient organizational form from the perspective of the downstream party. The objective of this section is to illustrate how the eventuality of collusion within firms and incongruent preferences between firms influence the characteristics of implemented transactions. For illustrative purposes, I demonstrate the potential effects for spot contracts and relational contracts separately. Finally, the observations are illustrated and conclusively discussed.

3.5.1 The Optimal Spot Contract

Suppose first that relational contracts are not feasible. This may occur if the interest rate is too high such that implicit contracts are not credible, or the involved parties can only commit to transactions lasting for a finite period of time. Accordingly, the downstream party can only select one of three spot contracts: spot employment, supervision, or spot market. The optimal spot contract eventually determines the downstream's best fall-back position for relational contracts and hence, their self-enforcement conditions.

The downstream's decision in terms of an integrated production or market transaction is conditional on three parameter values: (*i*) the spread of the internal good value ΔI, (*ii*) the spread of the external good value ΔE; and (*iii*), the supervisor's reservation utility $\bar{U}^S$. The congruity measure φ, however, affects the upstream's, but not the downstream's expected profit in a spot environment, see section 3.4.4. To identify the effect of collusive behavior on the downstream's optimal strategy, let us first consider the collusion-free case.

Proposition 3.5 *Consider the collusion-free case and suppose that only spot contracts are feasible. Then, the downstream party receives*

$$\Pi^D = \begin{cases} \Pi^{D|SM}, & \text{if } \; 0 < (\Delta E)^2 \leq \underline{\Theta} \\ \Pi^{D|S}, & \text{if } \; \underline{\Theta} < (\Delta E)^2, \end{cases} \tag{3.47}$$

where

$$\underline{\Theta} \equiv 2\left[2\bar{U}^S - I_L\right]. \tag{3.48}$$

Proof See appendix.

Apparently from above, there exists a threshold level of ΔE implying that the downstream party is indifferent between an integrated production and a market transaction. The downstream party prefers a mutual spot transaction with the upstream party if $(\Delta E)^2 \leq \underline{\Theta}$, and supervision, otherwise. In another sense, a market transaction is superior to an integrated production if the market valuation towards the difference between the low and high external good value is sufficiently low. This directly implies a lower bargaining position for the upstream party and hence, a lower transfer price. However, if $(\Delta E)^2 > \underline{\Theta}$, acquiring the good via a market transaction is more costly than an integrated production. In this case, it is more beneficial for the downstream party to recruit a supervisor aimed at providing the worker with explicit incentives.

Without side-contracting, supervision is found to be superior for sufficiently high spreads in the external good value. However, if we were to allow collusive behavior, it would always occur in a spot environment due to the lack of prospective punishments. Recall that the downstream party obtains only the spot employment profit under supervision if collusion is anticipated either by the worker or by the downstream party. As a result, the downstream's one-shot alternatives shrink to spot employment and spot market transactions.

Proposition 3.6 *Consider the collusion case and suppose that only spot contracts are feasible. Then, the downstream party receives*

$$\Pi^D = \begin{cases} \Pi^{D|SM}, & \text{if } 0 < (\Delta E)^2 \leq \overline{\Theta} \\ \Pi^{D|SE}, & \text{if } \overline{\Theta} < (\Delta E)^2, \end{cases} \tag{3.49}$$

where

$$\overline{\Theta} \equiv (\Delta I)^2 - 2I_L. \tag{3.50}$$

Proof See appendix.

Spot market transactions apparently remain optimal for sufficiently low spreads in the external good value ΔE. However, the downstream party now prefers spot market transactions for $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Theta}$ instead of employing a supervisor for an integrated production.[16] In this case, supervision cannot be profitable as a result of potential collusion between the supervisor and either the worker or the downstream party herself. To put it differently, the threat of collusive behavior forces the downstream party to use market instead of integrated transactions, which would have been more efficient otherwise. Utilizing the market for acquiring the good is the downstream's most efficient strategy to bypass collusion and the associated costs for $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Theta}$. However, things are different for $\overline{\Theta} < (\Delta E)^2$. Here, the downstream party prefers spot employment to bypass collusion. There are two reasons for this observation. First, supervision does not provide a higher expected profit due to vertical side-contracting. Second, the spread of the external good value and consequently, the transfer price as a result of the upstream's bargaining position, is too high such that the acquisition of the good via the market is less beneficial than an integrated spot production.

3.5.2 The Optimal Relational Contract

Now suppose that all parties interact infinitely such that relational contracts contingent on observable, but non-verifiable information can be credible. If the involved parties are better off by honoring a sustained relationship rather than by violating their implicit obligations, no party has incentives to deviate from the stipulated behavior and relational contracts are self-enforcing. In this case, the optimal transaction additionally depends on the interest rate r, which can be interpreted as the patience of all involved entities.

Suppose for a moment that the interest rate r is sufficiently low such that all considered implicit contracts with the efficient incentives schemes are self-enforcing. Formally, this requires $r \leq \min\{r^{RM}, r^{RE}\}$. Then, the downstream party prefers relational market if $\Pi^{D|RM} \geq \Pi^{D|RE}$, and relational employment, otherwise. I demonstrate in the appendix that $\Pi^{D|RM} \geq \Pi^{D|RE}$ requires

$$(\Delta E)^2 \leq \underline{\Phi}, \quad \text{if } \Pi^{D|SM} = \tilde{\Pi}^{RM} \tag{3.51}$$

$$(\Delta E)^2 \leq \overline{\Phi}, \quad \text{if } \Pi^{D|SM} \neq \tilde{\Pi}^{RM}, \tag{3.52}$$

where

$$\underline{\Phi} \equiv \left[\left(\kappa^2 + (\Delta I)^2 - 4I_L\right)^{\frac{1}{2}} - \kappa\right]^2 \qquad \overline{\Phi} \equiv \frac{1}{2}(\Delta I)^2 \qquad \kappa \equiv \Delta I \cos\varphi.$$

[16] Note that $\underline{\Theta} < \overline{\Theta}$ since $\bar{U}^S < (\Delta I)^2/4$.

A sustained bilateral market transaction with the upstream party (relational market) is more profitable if the spread in the external good value ΔE is sufficiently low. The rationale is that a lower ΔE diminishes the upstream's bargaining position for spot transactions, and therefore, mitigates the downstream's costs for ensuring her participation. Observe, however, that we have two different threshold levels $\underline{\Phi}$ and $\overline{\Phi}$. Recall that the downstream's expected profit for relational market is conditional on whether or not a spot market transaction with the upstream party is her best alternative. This leads to two different cut offs, where $\underline{\Phi}$ is relevant if spot market transactions are the downstream's best fall-back, and $\overline{\Phi}$ otherwise.

For $r^i < r \leq \widehat{r}^i$, $i = RM, RE$, the downstream party needs to adjust incentives for both relational contracts in order to ensure their self-enforcement. The downstream party thereby prefers relational market if $\Pi^{D|RM}(r) \geq \Pi^{D|RE}(r)$, and relational employment, otherwise. For the subsequent analysis, let $(\Delta E)^2 = \underline{\Psi}(r)$ be the squared spread of the external good value implying $\Pi^{D|RM}(r) = \Pi^{D|RE}(r)$ for $\tilde{\Pi}^{RM} = \Pi^{D|SM}$, and $(\Delta E)^2 = \overline{\Psi}(r)$ for $\tilde{\Pi}^{RM} \neq \Pi^{D|SM}$, respectively.[17] Then, $\Pi^{D|RM}(r) \geq \Pi^{D|RE}(r)$ requires

$$(\Delta E)^2 \leq \underline{\Psi}(r), \quad \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SM} \tag{3.53}$$

$$(\Delta E)^2 \leq \overline{\Psi}(r), \quad \text{if } \tilde{\Pi}^{RM} \neq \Pi^{D|SM}. \tag{3.54}$$

To identify the effect of collusion on the optimal relational contract and its self-enforcement condition, assume for a moment that side-contracting cannot occur. In this case, we know from proposition 3.5 that supervision is the superior spot contract for $(\Delta E)^2 > \underline{\Theta}$, and spot market transactions with the upstream party, otherwise.

Proposition 3.7 *Consider the collusion-free case. The downstream party implements relational market and receives $\Pi^{D|RM}(r)$ in the intervals*

$$\begin{cases} 0 < (\Delta E)^2 \leq \min\{\underline{\Phi}, \underline{\Theta}\} \ \textit{and} \ \underline{\Theta} < (\Delta E)^2 \leq \overline{\Phi}, & \textit{if } r \leq r^{RM}; \\ 0 < (\Delta E)^2 \leq \min\{\underline{\Psi}(r), \underline{\Theta}\} \ \textit{and} \ \underline{\Theta} < (\Delta E)^2 \leq \overline{\Psi}(r), & \textit{if } r^{RM} < r \leq \widehat{r}^{RM}, \end{cases}$$

where

$$r^{RM} = \begin{cases} \dfrac{1}{8\Delta I}\left[\Delta I \boldsymbol{\mu} - \Delta E \boldsymbol{\omega}\right]^2, & \textit{if } \ 0 < (\Delta E)^2 \leq \min\{\underline{\Phi}, \underline{\Theta}\} \\ \dfrac{\Delta I}{4} + \dfrac{1}{2\Delta I}\left[2\bar{U}^S - (\Delta E)^2\right], & \textit{if } \ \underline{\Theta} < (\Delta E)^2 \leq \overline{\Phi}, \end{cases} \tag{3.55}$$

and,

$$\widehat{r}^{RM} = \begin{cases} \Delta I - \left[\dfrac{1}{4}(\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega})^2 + \dfrac{1}{2}\left[(\Delta I)^2 - (\Delta E)^2\right]\right]^{\frac{1}{2}}, \\ \qquad\qquad\qquad\qquad\qquad\qquad \textit{if } \ 0 < (\Delta E)^2 \leq \min\{\underline{\Psi}(r), \underline{\Theta}\} \\ \Delta I - \left[(\Delta E)^2 + \dfrac{1}{2}(\Delta I)^2 - 2\bar{U}^S\right]^{\frac{1}{2}}, \quad \textit{if } \ \underline{\Theta} < (\Delta E)^2 \leq \overline{\Psi}(r). \end{cases} \tag{3.56}$$

[17]Due to the structure of $\Pi^{D|RM}(r)$ and $\Pi^{D|RE}(r)$ for $r^i < r \leq \widehat{r}^i$, $i = RM, RE$, one cannot achieve a tractable closed form solution. Nevertheless, using the implicit characterizations does not derogate the subsequent results.

In contrast, the downstream party implements relational employment and receives $\Pi^{D|RE}(r)$ *in the intervals*

$$\begin{cases} \underline{\Phi} < (\Delta E)^2 \leq \underline{\Theta} \;\; \textit{and} \;\; \overline{\Phi} < (\Delta E)^2, & \textit{if } r \leq r^{RE}; \\ \underline{\Psi}(r) < (\Delta E)^2 \leq \underline{\Theta} \;\; \textit{and} \;\; \overline{\Psi}(r) < (\Delta E)^2, & \textit{if } r^{RE} < r \leq \widehat{r}^{RE}, \end{cases}$$

where

$$r^{RE} = \begin{cases} \dfrac{1}{2\Delta I}\left[(\Delta E)^2 + 2I_L\right], & \textit{if } \;\underline{\Phi} < (\Delta E)^2 \leq \underline{\Theta} \\ \dfrac{2\bar{U}^S}{\Delta I}, & \textit{if } \;\overline{\Phi} < (\Delta E)^2, \end{cases} \tag{3.57}$$

and,

$$\widehat{r}^{RE} = \begin{cases} \Delta I + I_L + \dfrac{1}{2}\left[(\Delta E)^2 - (\Delta I)^2\right], & \textit{if } \;\underline{\Psi}(r) < (\Delta E)^2 \leq \underline{\Theta} \\ \Delta I - \dfrac{1}{2}(\Delta I)^2 + 2\bar{U}^S, & \textit{if } \;\overline{\Psi}(r) < (\Delta E)^2. \end{cases} \tag{3.58}$$

If in contrast $r > \widehat{r}^{RM}$ *or* $r > \widehat{r}^{RE}$ *in the relevant intervals, the downstream party receives* $\Pi^D = \max\{\Pi^{D|SM}, \Pi^{D|S}\}$.

Proof See appendix.

The cut off interest rates $\widehat{r}^{RM}$ and $\widehat{r}^{RE}$ determine whether or not the respective relational contract is credible. Suppose for a moment that both relational contracts are self-enforcing. Generally, relational market is superior for low values of ΔE. Nevertheless, relational employment can be temporarily preferred by the downstream party in the interval $\underline{\Psi}(r) < (\Delta E)^2 \leq \underline{\Theta}$, whereas for $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Psi}(r)$ relational market is again more profitable. The reason for obtaining spanned intervals is as follows. Different fall-back positions for relational market—spot market or integrated spot productions—impose diverse costs for ensuring the upstream's participation. This further provides the downstream party with different expected profits, and as a consequence, diverging self-enforcement conditions. Finally observe that the previous argumentation only applies if $\underline{\Psi}(r) < \underline{\Theta}$ and $\underline{\Theta} < \overline{\Psi}(r)$, which in turn depends on the specific parameter values. Otherwise, there exists only one threshold level of ΔE implying indifference between relational market and relational employment.

To shed more light on the derived self-enforcement conditions, one can show that (3.56) for $0 < (\Delta E)^2 \leq \min\{\underline{\Psi}(r), \underline{\Theta}\}$ is equivalent to

$$\widehat{r}^{RM} = \Delta I - \left[\frac{1}{4}\left[3(\Delta I)^2 - (\Delta E)^2 + 2\Delta I \Delta E \cos\varphi\right]\right]^{\frac{1}{2}}. \tag{3.59}$$

As discussed in section 3.4.5, $\widehat{r}^{RM}$ is increasing in the congruity measure φ, provided the downstream's next alternative is to utilize spot market transactions. Technically, this applies for the interval $(\Delta E)^2 \leq \min\{\underline{\Phi}, \underline{\Theta}\}$. It can be verified that $\widehat{r}^{RM}$ is convex decreasing in ΔE as long as $\Delta E \leq \Delta I \cos\varphi$, and increasing, otherwise. The same observation can be made for r^{RM}. An increasing spread of the external good value ΔE has two opposite effects. First, it imposes a higher floor payment

P_L aimed at ensuring the upstream's participation. This impairs the profitability of relational market. As a result, it becomes more beneficial for the downstream party to renege on $\Delta P^*(r)$ such that $\widehat{r}^{RM}$ can be expected to decrease. Second, a higher spread ΔE reduces the profitability of spot market transactions due to the upstream's enhanced bargaining position. Thus, it becomes less beneficial for the downstream party to renege, which should be reflected by a higher $\widehat{r}^{RM}$. As (3.59) indicates, the first effect outweighs the second as long as $\Delta E \leq \Delta I \cos\varphi$, and vice versa for $\Delta E > \Delta I \cos\varphi$. In the interval $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Psi}(r)$ where the downstream's alternative is an integrated production, $\widehat{r}^{RM}$ is always decreasing in ΔE. The same applies for r^{RM} in the interval $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Phi}$.

Finally observe that r^{RE} and $\widehat{r}^{RE}$ are increasing in ΔE for $\underline{\Phi} < (\Delta E)^2 \leq \underline{\Theta}$ and $\underline{\Psi}(r) < (\Delta E)^2 \leq \underline{\Theta}$, respectively. This obtains because the value of the downstream's alternative—utilizing spot market transactions with the upstream party—is decreasing in ΔE, thereby providing her less incentives to violate the implicit contract. For $\overline{\Phi} < (\Delta E)^2$, however, r^{RE} is constant in ΔE because relational employment, and supervision as best fall-back, are independent of ΔE. The same observation applies for $\widehat{r}^{RE}$ in the interval $\overline{\Psi}(r) < (\Delta E)^2$.

Subsequently, let us investigate the effect of side-contracting on the optimal relational contract and its self-enforcement condition. Keep in mind that the downstream party can achieve the collusion-free payoff if $r \leq \min\{r^W, r^D\}$.

Proposition 3.8 *Consider the collusion case. In comparison to the collusion-free case, the cut off interest rates for relational market change to*

$$r^{RM} = \begin{cases} \dfrac{1}{8\Delta I}\left[\Delta I\boldsymbol{\mu} - \Delta E\boldsymbol{\omega}\right]^2, & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SM} \\ \dfrac{1}{2\Delta I}\left[(\Delta I)^2 - (\Delta E)^2\right], & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SE}, \end{cases} \tag{3.60}$$

and,

$$\widehat{r}^{RM} = \begin{cases} \dfrac{1}{8\Delta I}\left[\Delta I\boldsymbol{\mu} - \Delta E\boldsymbol{\omega}\right]^2, & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SM} \\ \Delta I - \Delta E, & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SE}. \end{cases} \tag{3.61}$$

In contrast, the cut off interest rates for relational employment become

$$r^{RE} = \frac{\Delta I}{2} - \frac{2}{\Delta I}\left[\max\{\Pi^{D|SM}, \Pi^{D|SE}\} - I_L\right], \tag{3.62}$$

and,

$$\widehat{r}^{RE} = \Delta I - 2\left[\max\{\Pi^{D|SM}, \Pi^{D|SE}\} - I_L\right]^{\frac{1}{2}}. \tag{3.63}$$

Finally, supervision is not implemented for all r.

Proof See appendix.

There are two implications which are important to discuss. Firstly, we know that supervision is not collusion-proof if $r > \widehat{r}^S$. Instead of employing a supervisor, the downstream party is compelled to use either less valuable spot market transactions for $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Theta}$, or spot employment for $\overline{\Theta} < (\Delta E)^2$. Due to a higher prospective punishment for violating implicit agreements, the downstream party is less tempted to renege, which in turn is reflected by higher cut off interest rates for relational market and relational employment. The second important implication is that collusion-proof supervision is in neither case implemented. The rationale for this observation is that collusion-proof supervision requires a sufficiently low interest rate r which contemporaneously guarantees the self-enforcement of either relational market or relation employment with the efficient incentive schemes for the relevant intervals. In this case, the downstream party prefers one of the relational contracts instead of supervision.

3.5.3 Graphical Representation

I emphasized in preceding sections several conditions determining the optimal transactions and contractual arrangements from the perspective of the downstream party. The purpose of this section is to illustrate, how the optimal organizational form is (i), influenced by incongruent preferences between the downstream and upstream party; and (ii), affected by anticipated collusion between the supervisor and either the worker or the downstream party.

The optimal transactions and contractual arrangements are illustrated in figure 3.1, where the squared spread of the external good value $(\Delta E)^2$ is on the horizontal axis and the interest rate r on the vertical axis. This graphical representation applies for the case $\underline{\Phi}, \underline{\Psi}(r) > \underline{\Theta}$ and $\overline{\Psi}, \overline{\Psi}' \in \left(\underline{\Theta}, \overline{\Theta}\right)$, where $\overline{\Psi} \equiv \overline{\Psi}(\widehat{r}^{RM})$ for the collusion-free case, and $\overline{\Psi}' \equiv \overline{\Psi}(\widehat{r}^{RM})$ for the collusion case, respectively. Additionally, it is assumed that $(\Delta I)^2 \cos^2 \varphi > \underline{\Theta}$.[18] The subsequent explanations apply in general to the other cases as well.

First, I briefly discuss how diverging preferences between the downstream and upstream party for the characteristics of exchanged goods affect the optimal transaction. For the sake of parsimony, I restrict the subsequent explanations to the collusion-free case. Similar observations, however, apply also for the collusion case. Recall that more congruent preferences—characterized by a smaller φ—affect the downstream's payoffs in two ways: (i) it diminishes her expected profit for relational market, and therefore, (ii) mitigates her prospective punishment for reneging on the promised premium $\Delta P(r)$. Both effects are reflected by lower threshold interest rates r^{RM} and $\widehat{r}^{RM}$. The reversed is true if φ increases. Both effects, however, only apply, if spot market transactions are the downstream's best alternative. Graphically, a smaller φ causes the line AB in figure 3.1 to shift downwards (dotted line). Consider the area $T1$ below the line AB. Here, the downstream party could engage in a superior long-term relationship with the upstream party (relational market) for sufficiently incongruent production technologies (high φ). If in contrast the technology provides sufficiently congruent preferences, the downstream party is compelled to utilize less profitable spot transactions with the upstream party. Consequently, the downstream party cannot motivate the upstream party to make relation-specific investments. The analysis in section 3.4.5 additionally indicates that more congruent preferences diminish the profitability of relational employment for $r^{RM} < r \leq \widehat{r}^{RM}$.

[18]This condition implies that $\widehat{r}^{RM}$ is decreasing in $(\Delta E)^2$ for $0 < (\Delta E)^2 \leq \min\{\underline{\Psi}(r), \underline{\Theta}\}$.

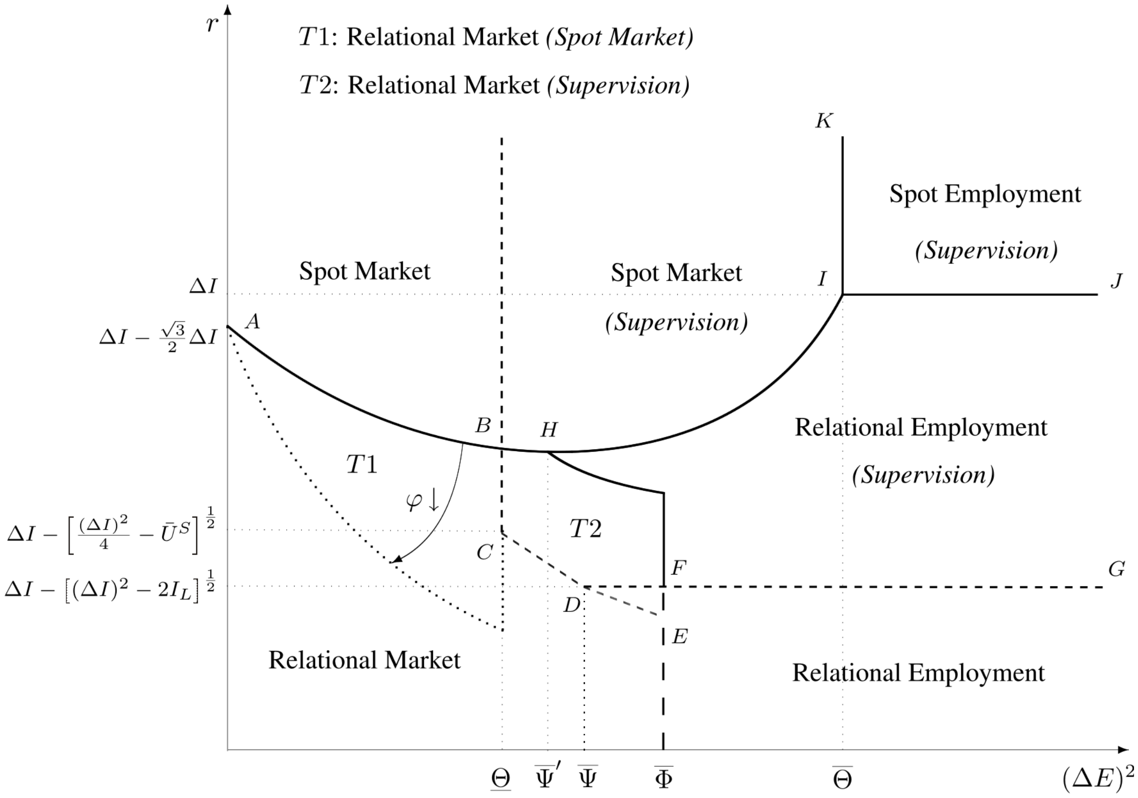

Figure 3.1: Optimal Transactions and Contracts

Next, let us turn to collusion and its effect on the optimal organizational form. In addition to the optimal transactions and contracts for the collusion case, figure 3.1 also exemplifies the downstream's optimal strategy for the collusion-free case (dashed lines, where the optimal strategies are parenthesized).[19] First consider the downstream's best spot transactions, which eventually determine her best fall-back positions for relational contracts. Spot contracts are chosen in the area above the lines AB; and additionally CDG for the non-collusion case, and BIJ for the collusion case, respectively. In these areas, the downstream party cannot credibly commit to honor implicit agreements such that relational contracts are not feasible. Obviously, if $(\Delta E)^2 \leq \underline{\Theta}$, i.e. the spread of the external good value is sufficiently low, a spot market transaction is always superior. In contrast, for moderate spreads characterized by $\underline{\Theta} < (\Delta E)^2 < \overline{\Theta}$, the downstream party prefers spot market transactions when collusion is anticipated, whereas supervision would have been more beneficial otherwise. Generally speaking, potential collusion forces the downstream party in this case to adopt less efficient market transactions instead of an integrated production. Finally, for high spreads with $\overline{\Theta} < (\Delta E)^2$, the downstream party prefers spot employment rather than employing a supervisor in order to prevent collusive behavior and the associated costs.

Potential collusion has a negative effect from the downstream's perspective, whenever employing a supervisor to achieve contractible information would be opti-

[19]The only exception is area $T1$ which refers to the exemplified effect of preference congruity and is not influenced by collusion.

mal, but the induced costs for preventing side-contracting compromise its profitability. Now consider the effect of anticipated collusion on relational contracts. We know from the previous section that worse fall-back positions as a result of anticipated collusion may facilitate superior relational contracts for higher interest rates. In the area $T2$ in figure 3.1, the downstream party can achieve a sustained relationship with the upstream party based on implicit agreements, and therefore, can motivate relation-specific investments, which would have not been feasible otherwise. Likewise, anticipated collusion enables the downstream party in the area between the lines HIJ and FG to provide the worker with a credible relational incentive contract. A further effect is the improvement of the downstream expected profit for $r^i < r \leq \widehat{r}^i$, $i = RM, RE$. This obtains because a worse fall-back position enables the downstream party to provide either the worker or the upstream party with a more efficient incentive scheme without violating the respective self-enforcement condition.

Summarizing the previous observations, more congruent preferences for the characteristics of exchanged goods—implied by the upstream's production technology—can be disadvantageous for the downstream party with respect to two dimensions: Firstly, it diminishes her expected profit in case she engages in a bilateral relationship with the upstream party based on implicit agreements, but the efficient incentive scheme is not credible ($r^{RM} < r \leq \widehat{r}^{RM}$). Secondly, it implies a lower cut off interest rate $\widehat{r}^{RM}$ such that for some r, relational market can not be facilitated anymore and the downstream party is forced to utilize less profitable spot market transactions. In contrast, anticipated collusion may have two opposite effects on the efficiency of transactions. First, if superior relational contracts are not feasible, potential collusion can force organizations to use either less profitable market or integrated transactions in order to bypass the required costs for preventing side-contracting. Second, collusion and its harmful effect on the efficiency of integrated productions can be advantageous for organizations if it facilitates the achievement, or enhances the profitability, of superior relational contracts. Generally speaking, collusion as an internal inefficiency can (i), improve the profitability of integrated and market transactions; and (ii), facilitate the achievement of superior long-term relationships based on non-enforceable agreements in place of less efficient spot transactions. This also suggests that anticipated collusion within firms may motivate or improve relation-specific investments in inter-firm trade.

3.6 Availability of Verifiable Signals

So far, I considered only the employment of a supervisor as a device for the downstream party to receive contractible information for providing the worker with incentives. The drawback of employing a supervisor is that preventing side-contracting imposes additional costs which compromise the efficiency of an integrated production. I believe it is worthwhile to briefly consider the effect on the optimal transaction if the downstream party receives verifiable but imperfect information about the worker's performance. For the sake of conciseness, I restrict the proceeding analysis and explanations to the collusion case.

Suppose the downstream party receives a binary and verifiable signal $Y \in \{0, 1\}$, where $Y = 1$ is the favorable signal in the sense of Milgrom [1981]. The probability of realizing the favorable signal is conditional on the worker's effort and takes the form

$$\text{Prob}[Y = 1|\boldsymbol{e}_W] = \min\{\boldsymbol{\tau}^t \boldsymbol{e}_W, 1\}, \tag{3.64}$$

where $\boldsymbol{\tau} = (\tau_1, ..., \tau_n)^t$, $\boldsymbol{\tau} \in \mathbb{R}^{n+}$. To ensure an interior solution, assume that $\boldsymbol{\tau}$ is characterized such that $\boldsymbol{\tau}^t \boldsymbol{e}_W < 1$ for the second-best effort vector.[20] The downstream part can now provide the worker with the bonus contract

$$w^W = \begin{cases} \alpha + \beta, & \text{if} \quad Y = 1 \\ \alpha, & \text{if} \quad Y = 0. \end{cases} \tag{3.65}$$

The optimal bonus contract therefore solves

$$\max_{\alpha, \beta, \boldsymbol{e}_W} \Pi^{D|SE} \equiv I_L + \Delta I \boldsymbol{\mu}^t \boldsymbol{e}_W - \alpha - \beta \boldsymbol{\tau}^t \boldsymbol{e}_W \tag{3.66}$$

s.t.

$$\alpha + \beta \boldsymbol{\tau}^t \boldsymbol{e}_W - \frac{1}{2} \boldsymbol{e}_W^t \boldsymbol{e}_W \geq 0 \tag{3.67}$$

$$\boldsymbol{e}_W \in \arg\max_{\tilde{\boldsymbol{e}}_W} \alpha + \beta \boldsymbol{\tau}^t \tilde{\boldsymbol{e}}_W - \frac{1}{2} \tilde{\boldsymbol{e}}_W^t \tilde{\boldsymbol{e}}_W \tag{3.68}$$

$$\alpha + \beta \geq 0 \tag{3.69}$$

$$\alpha \geq 0, \tag{3.70}$$

with (3.67) as the worker's participation, and (3.68) as her incentive constraint. Furthermore, (3.69) and (3.70) guarantee that the bonus contract is compatible with the worker's liability limit.

We can infer from the worker's incentive constraint that she chooses $\boldsymbol{e}_W = \beta \boldsymbol{\tau}$ in order to maximize her expected utility. In contrast to relational employment and supervision, her effort allocation depends now upon $\boldsymbol{\tau}$. Then, if there exists a constant $\lambda > 0$ satisfying $\boldsymbol{\tau} = \lambda \boldsymbol{\mu}$, the bonus contract on the basis of Y motivates the worker to implement the non-distorted (first-best) effort allocation. Otherwise, she inefficiently over- or underemphasizes some tasks relative to others in order to maximize the probability of obtaining the contracted bonus.

I demonstrate in the appendix that the worker's optimal bonus contract is characterized by

$$\alpha^* = 0, \qquad \beta^*(\theta) = \frac{\Delta I \cos\theta}{2\|\boldsymbol{\tau}\|},$$

where θ is the angle between vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\tau}$, and $\|\boldsymbol{\tau}\|$ denotes the length of $\boldsymbol{\tau}$.[21] The angle θ thereby quantifies how congruent the expected signal is with respect to the probability that the high internal good value will be realized. Since the expected signal manifests the worker's effort allocation, θ also characterizes the efficiency of her effort allocation relative to first-best.

[20] One can show by using the subsequently derived results that this requires $\|\boldsymbol{\tau}\| < \cos^{-1}\theta(\Delta I)^{-1}$, where θ denotes the angle between vector $\boldsymbol{\mu}$ and $\boldsymbol{\tau}$, and $\|\boldsymbol{\tau}\|$ the length of $\boldsymbol{\tau}$.

[21] Note that $\theta \in [0, \pi/2]$ since μ_i, $\tau_i > 0$, $i = 1, ..., n$. This implies $\beta^* \geq 0$.

By providing the worker with an explicit bonus contract contingent on Y, the downstream party receives

$$\Pi^{D|SE}(\theta) = I_L + \frac{1}{4}(\Delta I)^2 \cos^2 \theta. \tag{3.71}$$

The downstream's expected profit is a function of the congruity measure θ. To exemplify the particular effects of θ on $\Pi^{D|SE}(\theta)$, let us first consider the extreme cases. If $\theta = 0$, the signal is perfectly congruent. Then, the probability to produce the high internal good value is identical to the probability that the favorable signal will be realized. Accordingly, the downstream party can motivate the worker to implement the first-best effort allocation by providing her with a bonus contract contingent on the realization of Y. This further implies that the downstream party receives the same expected profit as under relational employment, but this is now feasible even in a spot environment. In this case, the downstream party chooses always a spot employment contract contingent on Y, unless ΔE and $r \leq \widehat{r}^{RM}$ are sufficiently small such that relational market provides her with a higher expected profit.

In contrast, if $\theta = \pi/2$, the signal does not provide any value since it would motivate the worker to implement an invaluable effort allocation. Formally, this implies $\text{Prob}\{I_i = I_H | \boldsymbol{e}_W, \theta = \pi/2\} = 0$.[22] Therefore, it is efficient from the downstream's perspective to set $\beta^* = 0$, thereby inducing the worker to implement $\boldsymbol{e}_W = (0, ..., 0)^t$. Accordingly, the availability of Y does not change the downstream's optimal strategy as illustrated in figure 3.1 if it is characterized by $\theta = \pi/2$.

Next, let us turn to the more interesting case with $0 < \theta < \pi/2$. The more congruent the signal becomes (θ decreases), the higher is the downstream's expected profit. To identify the effect on the optimal organizational form, consider first the downstream's preference for either utilizing a spot employment contract contingent on Y, or a spot market transaction with the upstream party. Standard calculations give

$$\Pi^{D|SE}(\theta) \geq \Pi^{D|SM} \quad \Leftrightarrow \quad (\Delta E)^2 \geq \overline{\Theta}(\theta) \equiv (\Delta I)^2 \left[1 - \cos^2 \theta\right] - 2I_L.$$

A more congruent signal (lower θ) reduces the threshold level $\overline{\Theta}(\theta)$, where the downstream party is indifferent between spot market transactions and utilizing an integrated production. Graphically, line IK in figure 3.1 shifts leftwards. Nevertheless, the value of Y and its effect on $\Pi^{D|SE}(\theta)$ influences the self-enforcement conditions for relational contracts, whenever spot employment is the downstream's best fallback position. Substituting $\Pi^{D|SE}(\theta)$ in $\widehat{r}^{RM}$ and $\widehat{r}^{RE}$ gives

$$\widehat{r}^{RM}(\theta) = \Delta I - \left[(\Delta E)^2 + \frac{1}{2}(\Delta I)^2 \cos^2 \theta\right]^{\frac{1}{2}}. \tag{3.72}$$

and,

$$\widehat{r}^{RE}(\theta) = \Delta I \left[1 - \cos \theta\right]. \tag{3.73}$$

[22] To see this, recall that $\boldsymbol{e}_W = \beta\boldsymbol{\tau}$. Consequently, $\text{Prob}\{I_i = I_H | \boldsymbol{e}_W\} = \beta\boldsymbol{\mu}^t\boldsymbol{\tau}$, which is equivalent to $\text{Prob}\{I_i = I_H | \boldsymbol{e}_W\} = \beta\tau^t\tau\cos\theta$. Thus, $\text{Prob}\{I_i = I_H | \boldsymbol{e}_W, \theta = \pi/2\} = 0$.

Observe that $\widehat{r}^{RM}(\theta)$ and $\widehat{r}^{RE}(\theta)$ are increasing in θ. To put it differently, a more congruent signal (smaller θ) requires a lower interest rate for relational contracts to be self-enforcing. Furthermore, a decreasing θ contemporaneously reduces the profitability of both relational contracts for $r^i < r \leq \widehat{r}^i$, $i = RM, RE$. This is because the downstream party needs to adjust the respective incentive scheme aimed at ensuring its self-enforcement. Accordingly, receiving a costless and sufficiently congruent signal can be disadvantageous from the downstream's perspective if it enhances the value of her respective fall-back position and thereby either compromises the viability of superior relational contracts, or diminishes their profitability. This obtains because the downstream party cannot credibly commit not to use the signal Y after reneging occurred.

3.7 Conclusion

The theory of the firm literature presents a variety of reasons on why certain firms utilize market instead of integrated transactions, and vice versa. This essay provides two supplementary rationales: the eventuality of collusive behavior within, and incongruent preferences for the characteristics of exchanged goods between firms.

The analysis in this essay points to the ambiguous effect of collusive behavior. First, collusion can be harmful for firms since it may force them to use less efficient market instead of integrated transactions. Second and less apparent, potential collusion can be beneficial whenever it deteriorates the efficiency of alternative integrated transactions such that superior relational contracts become either more likely to be feasible, or more profitable. This observation is also a theoretical underpinning of Tirole's [1988] argumentation that: "[...] potential collusion is often more important than actual collusion in understanding organizational behaviour"[p. 462].

Besides elaborating on the impact of collusive behavior on transactions, this essay further considers incongruent preferences between firms as a source of inefficiencies for inter-firm trade. Particularly, preferences are referred to be incongruent between firms, when their requirements for exchanged goods diverge due to potentially alternative usages. This eventually may lead to the exchange of less suitable goods from demanding firms' perspective due to either an inefficient level, or lack of relation-specific investments. As the analysis in this essay indicates, incongruent preferences play only a role for the efficiency of transactions if firms engage in mutual market transactions based on non-enforceable agreements. In this case, it is shown that more congruent preferences for the characteristics of exchanged goods are disadvantageous from the demanding firm's perspective since this imposes higher costs to maintain a long-term relationship with the supplying firm aimed at ensuring relation-specific investments.

This essay demonstrated in a short extension that organizations with access to sufficiently valuable performance measures are more likely to adopt integrated productions rather than utilizing the market. However, the availability of valuable performance measures can also be disadvantageous in that they deteriorate the feasibility, or the profitability, of relational contracts. Thus, we can observe the initially counter-intuitive situation where a better internal performance measurement impair the efficiency of implicit contracts within and between firms.

Despite the simplicity of the considered framework, I believe it provides powerful insights in the theory of the firm. This essay proposes a pertinent contribution in explaining why some firms choose to adopt market transactions, even if an internal production appears to be more efficient. The general observation is that potential collusion as an internal inefficiency, and diverging preferences betweens firms as the downside of market transactions can partially explain the characteristics of transactions within and between firms.

This essay offered a preliminary glimpse into the linkages between the theory of the firm and (*i*) collusive behavior within firms, and (*ii*), incongruent preferences for the properties of exchanged goods between firms. However, our understanding of their consequences on the characteristics of internal and market transactions is far from complete. For instance, I disregarded the competition among demanding as well as supplying firms on the market for goods. The degree of concentration in an industry directly affects the bargaining position of the market participants and consequently, the value of inter-firm trade. The investigation of how market characteristics influence organizations' endeavor to prevent internal collusion and affect relation-specific investments could contribute to our understanding of the nature of firms. However, I leave this issue for future investigations.

3.8 Appendix

Proof of Proposition 3.1.
First, it is necessary that $\beta > 0$ in order to ensure that the worker implements $e_{Wi} > 0$ for at least one $i \in \{1, ..., n\}$. Consequently, (3.10) is satisfied as long as (3.11) holds and therefore omits. Assume for a moment that (3.12) is satisfied for the optimal bonus contract. Recall that $\boldsymbol{\mu}^t\boldsymbol{\mu} = \|\boldsymbol{\mu}\|^2 = 1$. Then, the Lagrangian becomes

$$\mathcal{L}(\alpha, \beta) = I_L + \Delta I \beta - \alpha - \beta^2 + \lambda \left[\alpha + \frac{1}{2}\beta^2\right] + \xi\alpha. \tag{3.74}$$

The first-order conditions are

$$-1 + \lambda + \xi = 0, \tag{3.75}$$

$$\Delta I + \beta\,(\lambda - 2) = 0, \tag{3.76}$$

and the complementary slackness conditions,

$$\lambda \left[\alpha + \frac{1}{2}\beta^2\right] = 0, \tag{3.77}$$

$$\xi\alpha = 0. \tag{3.78}$$

To find a solution of this problem, suppose for a moment that $\lambda > 0$. Accordingly, $\alpha + \beta^2/2 = 0$ due to (3.77). Since $\alpha \geq 0$, this would imply that $\alpha^* = 0$ and $\beta^* = 0$, and consequently, $\boldsymbol{e}^* = (0, ..., 0)^t$. Hence, $\lambda > 0$ cannot be a solution of this problem. Consequently, $\lambda = 0$, i.e. the worker's participation constraint is not binding. Furthermore, we can infer from (3.75) that $\xi = 1$. Then, (3.78) implies that $\alpha^* = 0$. Re-arranging (3.76) with $\lambda = 0$ gives $\beta^* = \Delta I/2$. This eventually leads to $\Pi^{D|RE} = I_L + (\Delta I)^2/4$. Substituting $\Pi^{D|RE}$ and $\beta^* = \Delta I/2$ in (3.12) gives the cut off interest rate

$$r \leq r^{RE} \equiv \frac{\Delta I}{2} - \frac{2}{\Delta I}\left[\tilde{\Pi}^{RE} - I_L\right]. \tag{3.79}$$

If $r > r^{RE}$, the optimal bonus contract violates (3.12). Hence, the downstream party needs to adjust β such that (3.12) is satisfied for $r > r^{RE}$. To identify the direction of the adjustment, define $F \equiv I_L + (\Delta I - r)\beta - \beta^2 - \tilde{\Pi}^{RE}$. Consequently, the Implicit Function Theorem gives $d\beta/dr = -(\partial F/\partial r)/(\partial F/\partial \beta)$, which leads to

$$\frac{d\beta}{dr} = \frac{\beta}{\Delta I - r - 2\beta}. \tag{3.80}$$

Notice that $d\beta/dr < 0$ in $\beta^* = \Delta I/2$. Thus, the downstream party needs to reduce β^*. Then, the downstream part chooses the maximum feasible β for $r > r^{RE}$ which satisfies (3.12). Observe further that is optimal to choose the lowest feasible α. Hence, $\alpha^* = 0$. We can infer from (3.12) that β^* solves

$$\beta^2 - (\Delta I - r)\beta - I_L + \tilde{\Pi}^{RE} \leq 0. \tag{3.81}$$

Applying the quadratic formula gives

$$\beta^*(r) = \frac{1}{2}(\Delta I - r) \pm \left[\frac{1}{4}(\Delta I - r)^2 + I_L - \tilde{\Pi}^{RE}\right]^{\frac{1}{2}}. \tag{3.82}$$

Since it is optimal to choose the highest feasible $\beta(r)$, the upper bound is the relevant one. Notice that there exits only a solution for $\beta(r)$ if $(\Delta I - r)^2/4 + I_L - \tilde{\Pi}^{RE} \geq 0$, which is equivalent to

$$r \leq \widehat{r}^{RE} \equiv \Delta I \pm 2\left[\tilde{\Pi}^{RE} - I_L\right]^{\frac{1}{2}}. \tag{3.83}$$

Observe that the upper bound leads to $\beta < 0$, which cannot be a solution. Accordingly, the lower bound determines $\widehat{r}^{RE}$. As a result, $\beta(r) = 0$ if $r > \widehat{r}^{RE}$. For parsimony, let

$$\phi \equiv \left[\frac{1}{4}\left(\Delta I - r\right)^2 + I_L - \tilde{\Pi}^{RE}\right]^{\frac{1}{2}}. \tag{3.84}$$

Substituting $\beta^*(r)$ in the downstream's objective function gives

$$\Pi^{D|RE}(r) = I_L + \frac{\Delta I}{2}(\Delta I - r) + \Delta I \phi - \frac{1}{4}(\Delta I - r)^2 - (\Delta I - r)\phi - \phi^2, \tag{3.85}$$

which is equivalent to

$$\Pi^{D|RE}(r) = I_L + \frac{\Delta I}{2}(\Delta I - r) - \frac{1}{4}(\Delta I - r)^2 + r\phi - \phi^2. \tag{3.86}$$

Substituting ϕ and re-arranging eventually lead to

$$\Pi^{D|RE}(r) = \frac{r}{2}\left[\Delta I - r + 2\left[\frac{1}{4}\left(\Delta I - r\right)^2 + I_L - \tilde{\Pi}^{RE}\right]^{\frac{1}{2}}\right] + \tilde{\Pi}^{RE}. \tag{3.87}$$

Recall that $\beta^* = 0$ if $r > \widehat{r}^{RE}$. Since this induces $\boldsymbol{e}_W^* = (0, ..., 0)^t$, it leads to $\Pi^{D|RE}(r) = I_L$.

Q.E.D.

Proof of Proposition 3.2.
Consider condition (3.20) which ensures that the supervisor does not collude. Since $w^S = \bar{U}^S$, the supervisor would always collude if either the worker or the downstream party offers to pay $T^i > 0$, $i = W, D$. Consequently, supervision can be only collusion-proof if neither the worker nor the downstream party is better off by engaging in side-contracting. The worker has no incentives to collude if $\bar{T}^W \leq 0$. We can see from (3.21) that this is equivalent to

$$r \leq \frac{EU^W}{\beta} \equiv r^W. \tag{3.88}$$

Substituting $\boldsymbol{e}_W^*$, α^* and β^* gives $r \leq \Delta I/4 \equiv r^W$. Similarly, the downstream party has no incentives to collude if $\bar{T}^D \leq 0$. From (3.22) we can observe that $\bar{T}^D \leq 0$ requires

$$r \leq \frac{\Pi^{D|S} - \tilde{\Pi}^S}{\beta} \equiv r^D. \tag{3.89}$$

Substituting $\boldsymbol{e}_W^*$, α^* and β^* leads to

$$r \leq \frac{\Delta I}{2} - \frac{2}{\Delta I}\left[\tilde{\Pi}^S + \bar{U}^S - I_L\right] \equiv r^D. \tag{3.90}$$

Thus, the optimal contracts are collusion-proof if $r \leq \min\{r^W, r^D\}$.
Now suppose $r > \min\{r^W, r^D\}$. Condition (3.20) implies that the supervisor refuses T^i if

$$T^i \leq \frac{1}{r}\left[w^S - \bar{U}^S\right]. \tag{3.91}$$

At a first glance, the downstream party could raise w^S aimed at ensuring that the supervisor refuses to accept $\bar{T}^i$, $i = W, D$. Note that enhancing w^S has a second effect on the downstream's incentive to collude. To see this, recall that $\Pi^{D|S} = \mathrm{E}[I_i - w^W - w^S]$. Enhancing w^S contemporaneously reduces $\Pi^{D|S}$ and thus, raises $\bar{T}^D$. As a consequence, enhancing w^S cannot prevent collusion.
Next, consider the case $r > r^W$. Observe from (3.88) that this implies

$$r \leq \frac{1}{\beta}\left[\alpha + \frac{1}{2}\beta^2\right]. \tag{3.92}$$

As for the supervisor's payment w^S, setting $\alpha > 0$ contemporaneously reduces $\Pi^{D|S}$ and thus, raises $\bar{T}^D$. Therefore, $\alpha^* = 0$, which implies $r \leq \beta/2$. We know that this condition is binding for $\beta^* = \Delta I/2$ in $r = r^W$. Thus, the downstream party needs to set $\beta^*(r) = 2r$ for $r > r^W$ in order to detain the worker from side-contracting. Substituting $\beta^*(r)$ in the downstream's objective function yields

$$\Pi^{D|S}(r) = I_L + 2r(\Delta I - 2r) - \bar{U}^S. \tag{3.93}$$

Now consider the case $r > r^D$. Observe from (3.89) that the downstream party does not collude if

$$0 \leq I_L + (\Delta - r)\beta - \beta^2 - \bar{U}^S - \tilde{\Pi}^S. \tag{3.94}$$

Recall that this condition is binding for $\beta^* = \Delta I/2$ in $r = r^D$. Hence, the downstream needs to adjust β for $r > r^D$. To identify the direction of this adjustment, define $F \equiv I_L + (\Delta - r)\beta - \beta^2 - \bar{U}^S - \tilde{\Pi}^S$. The Implicit Function Theorem gives $d\beta/dr = -(\partial F/\partial r)/(\partial F/\partial \beta)$, and consequently,

$$\frac{d\beta}{dr} = \frac{\beta}{\Delta I - r - 2\beta}, \tag{3.95}$$

which is strictly negative in $\beta^* = \Delta I/2$. Thus, the downstream party needs to reduce β^*. Then, the downstream part chooses the highest feasible β for $r > r^D$, which satisfies (3.94). Applying the quadratic formula gives

$$\beta^*(r) = \frac{1}{2}(\Delta I - r) \pm \left[\frac{1}{4}(\Delta I - r)^2 + I_L - \bar{U}^S - \tilde{\Pi}^{RE}\right]^{\frac{1}{2}}. \tag{3.96}$$

Since it is optimal to choose the highest $\beta(r)$, the upper value of $\beta^*(r)$ is relevant. To derive the downstream's expected profit, let

$$\phi \equiv \left[\frac{1}{4}(\Delta I - r)^2 + I_L - \bar{U}^S - \tilde{\Pi}^{RE}\right]^{\frac{1}{2}}. \tag{3.97}$$

Substituting $\beta^*(r)$ in the downstream's objective function gives

$$\Pi^{D|S}(r) = I_L + \frac{\Delta I}{2}(\Delta I - r) + \Delta I\phi - \bar{U}^S - \frac{1}{4}(\Delta I - r)^2 - (\Delta I - r)\phi - \phi^2, \quad (3.98)$$

which is equivalent to

$$\Pi^{D|S}(r) = I_L + \frac{\Delta I}{2}(\Delta I - r) - \bar{U}^S - \frac{1}{4}(\Delta I - r)^2 + r\phi - \phi^2. \quad (3.99)$$

Substituting ϕ and re-arranging lead to

$$\Pi^{D|S}(r) = \frac{r}{2}\left[\Delta I - r + 2\left[\frac{1}{4}(\Delta I - r)^2 + I_L - \bar{U}^S - \tilde{\Pi}^{RE}\right]^{\frac{1}{2}}\right] + \tilde{\Pi}^{RE}. \quad (3.100)$$

Recall that collusion-proofness requires to enhance $\beta(r)$ if $r > r^W$; and to reduce $\beta(r)$ if $r > r^D$. Yet, there are two possible cases: $r^W < r^D$ and $r^W \geq r^D$. In the first case, $\beta^*(r)$ increases in r thereby enhancing r^W, but contemporaneously reducing r^D. In the second case, $\beta^*(r)$ decreases in r, thereby implying that r^D increases and r^W decreases. In both cases, the adjustment of $\beta^*(r)$ eventually implies that $\widehat{r}^S \equiv r^W(\bar{\beta}) = r^D(\bar{\beta})$. For $r > \widehat{r}^S$, however, collusion-proofness cannot be achieved. The cut off interest rate $\widehat{r}^S$ therefore implies

$$\frac{1}{\bar{\beta}}\left[I_L + \Delta I\bar{\beta} - \bar{\beta}^2 - \bar{U}^S - \tilde{\Pi}^S\right] = \frac{\bar{\beta}}{2}. \quad (3.101)$$

Re-arranging yields

$$\bar{\beta}^2 - \frac{2}{3}\Delta I\bar{\beta} - \frac{2}{3}\left[I_L - \bar{U}^S - \tilde{\Pi}^S\right] = 0. \quad (3.102)$$

Solving for $\bar{\beta}$ by applying the quadratic formula gives

$$\bar{\beta}_{(-)} = \frac{\Delta I}{3} - \left[\frac{1}{3^2}(\Delta I)^2 + \frac{2}{3}\left(I_L - \bar{U}^S - \tilde{\Pi}^S\right)\right]^{\frac{1}{2}}, \quad (3.103)$$

and,

$$\bar{\beta}_{(+)} = \frac{\Delta I}{3} + \left[\frac{1}{3^2}(\Delta I)^2 + \frac{2}{3}\left(I_L - \bar{U}^S - \tilde{\Pi}^S\right)\right]^{\frac{1}{2}}. \quad (3.104)$$

If $r^W < r^D$, the downstream party chooses the lowest feasible $\beta(r)$ such that $\bar{\beta}_{(-)}$ is relevant. For $r^D \leq r^W$, however, it is optimal to choose the highest feasible $\beta(r)$ such that $\bar{\beta}_{(+)}$ is relevant. Accordingly, we achieve $\widehat{r}^S$ by calculating $\widehat{r}^S = r^W(\bar{\beta}_{(-)})$, which is identical to $\widehat{r}^S = r^D(\bar{\beta}_{(+)})$. Since $r^W = \beta/2$, we obtain $\widehat{r}^S = \bar{\beta}_{(-)}/2$.

Q.E.D.

Proof of Proposition 3.3.
Since $\tilde{\Pi}^{RM} = \Pi^{D|SM}$, it follows $\bar{\Pi}^U = I_L/2 + [\Delta I\boldsymbol{\mu} + \Delta E\boldsymbol{\omega}]^2/8$. The downstream party sets P_L such that the upstream's participation constraint is binding. Solving for P_L and substituting $\boldsymbol{e}_U^* = \Delta P\boldsymbol{\mu}$ give

$$P_L = \frac{1}{2}I_L + \frac{1}{8}(\Delta I\boldsymbol{\mu} + \Delta E\boldsymbol{\omega})^2 - \frac{1}{2}(\Delta P)^2. \quad (3.105)$$

Suppose for a moment that (3.39) is satisfied for the optimal premium. Substituting P_L and $\boldsymbol{e}_U^*$ in the downstream's objective function yield the following simplified problem:

$$\max_{\Delta P} \Pi^{D|RM} \equiv \frac{1}{2} I_L + \Delta I \Delta P - \frac{1}{8} (\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega})^2 - \frac{1}{2} (\Delta P)^2. \tag{3.106}$$

The first derivative leads to $\Delta P^* = \Delta I$. Hence, the downstream party receives

$$\Pi^{D|RM} = \frac{1}{2} I_L + \frac{1}{2} (\Delta I)^2 - \frac{1}{8} [\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega}]^2. \tag{3.107}$$

Substituting $\Pi^{D|RM}$ and $\Delta P^* = \Delta I$ in (3.39) eventually gives

$$r \leq r^{RM} \equiv \frac{1}{8 \Delta I} [\Delta I \boldsymbol{\mu} - \Delta E \boldsymbol{\omega}]^2. \tag{3.108}$$

If $r > r^{RM}$, the optimal premium violates (3.39). As a consequence, the downstream party needs to adjust ΔP such that (3.39) is satisfied for $r > r^{RM}$. In order to identify the direction of the adjustment, substitute $\boldsymbol{e}_U^* = \Delta P \boldsymbol{\mu}$ and $\tilde{\Pi}^{RM} = \Pi^{D|SM}$ in (3.39), which yields after re-arranging

$$F \equiv I_L/2 + (\Delta I - r) \Delta P - (\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega})^2/8 - (\Delta P)^2/2 - \Pi^{D|SM}. \tag{3.109}$$

The Implicit Function Theorem gives $d\Delta P/dr = -(\partial F/\partial r)/(\partial F/\partial \Delta P)$, and consequently,

$$\frac{d\Delta P}{dr} = \frac{\Delta P}{\Delta I - r - \Delta P}. \tag{3.110}$$

Observe that $d\Delta P/dr < 0$ in $\Delta P^* = \Delta I$. Thus, the downstream party needs to reduce ΔP^*. Consequently, the downstream part chooses the highest feasible ΔP for $r > r^{RM}$ which satisfies (3.39). From (3.39) we can see that ΔP^* solves

$$(\Delta P)^2 - 2(\Delta I - r) \Delta P + \frac{1}{4} (\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega})^2 - I_L + 2\Pi^{D|SM} \leq 0. \tag{3.111}$$

Applying the quadratic formula gives

$$\Delta P^*(r) = \Delta I - r \pm \left[(\Delta I - r)^2 - \frac{1}{4} (\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega})^2 + I_L - 2\Pi^{D|SM} \right]^{\frac{1}{2}}. \tag{3.112}$$

Since it is optimal to choose the highest feasible $\Delta P(r)$, the upper bound is relevant. Moreover, there exits only a solution for $\Delta P(r)$ if

$$(\Delta I - r)^2 - \frac{1}{4} (\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega})^2 + I_L - 2\Pi^{D|SM} \geq 0. \tag{3.113}$$

Re-arranging leads to

$$r \leq \hat{r}^{RM} \equiv \Delta I \pm \left[\frac{1}{4} (\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega})^2 + 2\Pi^{D|SM} - I_L \right]^{\frac{1}{2}}. \tag{3.114}$$

Observe that the upper bound leads to $\Delta P < 0$, which cannot be a solution. Thus, the lower bound of $\widehat{r}^{RM}$ is relevant. As a result, $\Delta P^* = 0$ if $r > \widehat{r}^{RM}$. For parsimony, let

$$\phi \equiv \left[(\Delta I - r)^2 - \frac{1}{4} \left(\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega} \right)^2 + I_L - 2\Pi^{D|SM} \right]^{\frac{1}{2}}. \tag{3.115}$$

Substituting $\Delta P^*(r)$ in the downstream's objective function gives

$$\begin{aligned} \Pi^{D|RM}(r) &= \frac{1}{2} I_L + \Delta I(\Delta I - r) + \Delta I \phi - \frac{1}{2}(\Delta I - r)^2 - (\Delta I - r)\phi \\ &\quad - \frac{1}{2}\phi^2 - \frac{1}{8} \left(\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega} \right)^2, \end{aligned} \tag{3.116}$$

which can be re-arranged to

$$\Pi^{D|RM}(r) = \frac{1}{2} I_L + \Delta I(\Delta I - r) - \frac{1}{2}(\Delta I - r)^2 + r\phi - \frac{1}{2}\phi^2 - \frac{1}{8} \left(\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega} \right)^2. \tag{3.117}$$

Substituting ϕ eventually gives

$$\begin{aligned} \Pi^{D|RM}(r) &= r \left[\Delta I - r + \left[(\Delta I - r)^2 - \frac{1}{4} \left(\Delta I \boldsymbol{\mu} + \Delta E \boldsymbol{\omega} \right)^2 + I_L - 2\Pi^{D|SM} \right]^{\frac{1}{2}} \right] \\ &\quad + \Pi^{D|SM}. \end{aligned} \tag{3.118}$$

Finally, if $r > \widehat{r}^{RM}$, the principal sets $\Delta P^* = 0$. Then, it is also optimal to set $P_L^* = 0$ and to engage in spot market transactions. Hence, $\Pi^{D|RM}(r) = \Pi^{D|SM}$ for $r > \widehat{r}^{RM}$.

Q.E.D.

Proof of Proposition 3.4.
Since $\tilde{\Pi}^{RM} \neq \Pi^{D|SM}$, it follows $\bar{\Pi}^U = (\Delta E)^2/2$. The downstream party sets P_L such that the upstream's participation constraint is binding. By substituting $\boldsymbol{e}_U^* = \Delta P \boldsymbol{\mu}$ in (3.37) and solving for P_L, one get

$$P_L = \frac{1}{2}(\Delta E)^2 - \frac{1}{2}(\Delta P)^2. \tag{3.119}$$

Suppose for a moment that (3.39) is satisfied for the optimal premium. Hence, we can substitute P_L and $\boldsymbol{e}_U^*$ in the downstream's objective function and achieve the simplified problem

$$\max_{\Delta P} \Pi^{D|RM} \equiv I_L + \Delta I \Delta P - \frac{1}{2}(\Delta E)^2 - \frac{1}{2}(\Delta P)^2. \tag{3.120}$$

The first derivative leads to $\Delta P^* = \Delta I$. Consequently, the downstream party receives

$$\Pi^{D|RM} = I_L + \frac{1}{2} \left[(\Delta I)^2 - (\Delta E)^2 \right]. \tag{3.121}$$

Substituting $\Pi^{D|RM}$ and $\Delta P^* = \Delta I$ in (3.39) gives

$$r \le r^{RM} \equiv \frac{1}{2\Delta I} \left[2I_L + (\Delta I)^2 - (\Delta E)^2 - 2\tilde{\Pi}^{RM} \right]. \tag{3.122}$$

If $r > r^{RM}$, $\Delta P^* = \Delta I$ violates (3.39). Hence, the downstream party needs to adjust ΔP such that (3.39) is also satisfied for $r > r^{RM}$. In order to identify the direction of the adjustment, one can substitute $\boldsymbol{e}_U^*$ in (3.39) which yields after re-arranging $F \equiv I_L + (\Delta I - r)\Delta P - (\Delta P)^2/2 - (\Delta E)^2/2 - \tilde{\Pi}^{RM}$. The Implicit Function Theorem gives $d\Delta P/dr = -(\partial F/\partial r)/(\partial F/\partial \Delta P)$, and consequently,

$$\frac{d\Delta P}{dr} = \frac{\Delta P}{\Delta I - r - \Delta P}. \tag{3.123}$$

Notice that $d\Delta P/dr < 0$ in $\Delta P^* = \Delta I$. Thus, the downstream party needs to reduce ΔP^*. Therefore, the downstream part chooses the highest feasible ΔP for $r > r^{RM}$ which satisfies (3.39). We can infer from (3.39) that ΔP^* solves

$$(\Delta P)^2 - 2(\Delta I - r)\Delta P - 2I_L + (\Delta E)^2 + 2\tilde{\Pi}^{RM} \leq 0. \tag{3.124}$$

Applying the quadratic formula gives

$$\Delta P^*(r) = \Delta I - r \pm \left[(\Delta I - r)^2 + 2I_L - (\Delta E)^2 - 2\tilde{\Pi}^{RM}\right]^{\frac{1}{2}}. \tag{3.125}$$

Since it is optimal to choose the highest feasible $\Delta P(r)$, the upper bound is relevant. Furthermore, there exits only a solution for $\Delta P(r)$ if

$$(\Delta I - r)^2 + 2I_L - (\Delta E)^2 - 2\tilde{\Pi}^{RM} \geq 0. \tag{3.126}$$

Solving for r yields

$$r \leq \widehat{r}^{RM} \equiv \Delta I \pm \left[(\Delta E)^2 - 2I_L + 2\tilde{\Pi}^{RM}\right]^{\frac{1}{2}}. \tag{3.127}$$

The upper bound of $\widehat{r}^{RM}$ leads to $\Delta P < 0$, which cannot be a solution. Hence, the lower bound of $\widehat{r}^{RM}$ is relevant. As a result, $\Delta P^* = 0$ if $r > \widehat{r}^{RM}$. Next, let

$$\phi \equiv \left[(\Delta I - r)^2 + 2I_L - (\Delta E)^2 - 2\tilde{\Pi}^{RM}\right]^{\frac{1}{2}}. \tag{3.128}$$

Substituting $\Delta P^*(r)$ in the downstream's objective function gives

$$\Pi^{D|RM}(r) = I_L + \Delta I(\Delta I - r) + \Delta I\phi - \frac{1}{2}(\Delta E)^2 - \frac{1}{2}(\Delta I - r)^2 - (\Delta I - r)\phi - \frac{1}{2}\phi^2, \tag{3.129}$$

which is equivalent to

$$\Pi^{D|RM}(r) = I_L + \Delta I(\Delta I - r) - \frac{1}{2}(\Delta E)^2 - \frac{1}{2}(\Delta I - r)^2 + r\phi - \frac{1}{2}\phi^2. \tag{3.130}$$

Substituting ϕ and re-arranging yield

$$\Pi^{D|RM}(r) = r\left[\Delta I - r + \left[(\Delta I - r)^2 + 2I_L - (\Delta E)^2 - 2\tilde{\Pi}^{RM}\right]^{\frac{1}{2}}\right] - (\Delta E)^2 + \tilde{\Pi}^{RM}. \tag{3.131}$$

As shown, the downstream party sets $\Delta P^* = 0$ if $r > \widehat{r}^{RM}$. Then, it is also optimal to set $P_L^* = 0$ and to engage in spot market transactions. Hence, $\Pi^{D|RM}(r) = \Pi^{D|SM}$ for $r > \widehat{r}^{RM}$.

Q.E.D.

Proof of Proposition 3.5.
The downstream party can choose between spot employment, spot market transactions, and supervision. In the collusion-free case, $\Pi^{D|S} > \Pi^{D|SE}$ if $\bar{U}^S < (\Delta I)^2/4$, which is assumed to be satisfied, see section 3.4.3. Thus, it remains to identify the cut off $\underline{\Theta}$ implying $\Pi^{D|S} \leq \Pi^{D|SM}$ for every $(\Delta E)^2 \leq \underline{\Theta}$. Equivalently,

$$I_L + \frac{1}{4}(\Delta I)^2 - \bar{U}^S \leq \frac{1}{2}I_L + \frac{1}{4}\left[(\Delta I)^2 - (\Delta E)^2\right], \tag{3.132}$$

which can be re-arranged to $(\Delta E)^2 \leq 2\left[2\bar{U}^S - I_L\right] \equiv \underline{\Theta}$.

Q.E.D.

Proof of Proposition 3.6.
We know that collusion always occurs in a spot environment such that $\Pi^{D|S} = \Pi^{D|SE}$. Now, it remains to identify the threshold $\overline{\Theta}$ implying $\Pi^{D|SE} \leq \Pi^{D|SM}$ for every $(\Delta E)^2 \leq \overline{\Theta}$. Equivalently,

$$I_L \leq \frac{1}{2}I_L + \frac{1}{4}\left[(\Delta I)^2 - (\Delta E)^2\right]. \tag{3.133}$$

Solving for $(\Delta E)^2$ gives $(\Delta E)^2 \leq (\Delta I)^2 - 2I_L \equiv \overline{\Theta}$.

Q.E.D.

Comparison of Relational Market and Relational Employment.
Consider first the case $\tilde{\Pi}^{RM} = \Pi^{D|SM}$. Then, $\Pi^{D|RM} \geq \Pi^{D|RE}$ is equivalent to

$$\frac{1}{2}I_L + \frac{1}{2}(\Delta I)^2 - \frac{1}{8}\left[\Delta I\boldsymbol{\mu} + \Delta E\boldsymbol{\omega}\right]^2 \geq I_L + \frac{1}{4}(\Delta I)^2. \tag{3.134}$$

Standard calculations give

$$(\Delta E)^2 + 2\Delta I\cos\varphi\Delta E + 4I_L - (\Delta I)^2 \leq 0. \tag{3.135}$$

First, we can treat (3.135) as an equality. By applying the quadratic formula, one obtain

$$\Delta E = -\Delta I\cos\varphi \pm \sqrt{(\Delta I)^2\cos^2\varphi + (\Delta I)^2 - 4I_L}. \tag{3.136}$$

Observe that the upper bound is relevant since $\Delta E > 0$. Thus, $\Pi^{D|RM} \geq \Pi^{D|RE}$ requires

$$\Delta E \leq -\Delta I\cos\varphi + \sqrt{(\Delta I)^2\cos^2\varphi + (\Delta I)^2 - 4I_L}, \tag{3.137}$$

which is equivalent to

$$(\Delta E)^2 \leq \left[\left((\Delta I)^2\cos^2\varphi + (\Delta I)^2 - 4I_L\right)^{\frac{1}{2}} - \Delta I\cos\varphi\right]^2 \equiv \underline{\Phi}. \tag{3.138}$$

Next, consider the case $\tilde{\Pi}^{RM} \neq \Pi^{D|SM}$. In this case, $\Pi^{D|RM} \geq \Pi^{D|RE}$ is equivalent to

$$I_L + \frac{1}{2}\left[(\Delta I)^2 - (\Delta E)^2\right] \geq I_L + \frac{1}{4}(\Delta I)^2. \tag{3.139}$$

Re-arranging gives $(\Delta E)^2 \leq (\Delta I)^2/2 \equiv \overline{\Phi}$.

Q.E.D.

Proof of Proposition 3.7.
Suppose first that $r \leq r^i$, $i = RM, RE$. Then, it is necessary to identify whether the downstream party prefers both relational contracts in the same interval for different values of r. First, consider the intervals $0 < (\Delta E)^2 \leq \underline{\Phi}$ and $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Phi}$, where $\Pi^{D|RM} \geq \Pi^{D|RE}$. Suppose first that $r^{RM} \geq r^{RE}$, which is equivalent to

$$\frac{\Pi^{D|RM} - \tilde{\Pi}^{RM}}{\Delta I} \geq \frac{2\left[\Pi^{D|RE} - \tilde{\Pi}^{RE}\right]}{\Delta I}. \tag{3.140}$$

If $r^{RM} \geq r^{RE}$, we have $\tilde{\Pi}^{RM} = \max\{\Pi^{D|SM}, \Pi^{D|S}\}$ and $\tilde{\Pi}^{RE} = \Pi^{D|RM}$, where the particular value of $\tilde{\Pi}^{RM}$ depends on the respective interval. Then, $r^{RM} \geq r^{RE}$ is equivalent to

$$\Pi^{D|RM} - \max\{\Pi^{D|SM}, \Pi^{D|S}\} \geq 2\Pi^{D|RE} - 2\Pi^{D|RM}, \tag{3.141}$$

and consequently,

$$3\Pi^{D|RM} \geq 2\Pi^{D|RE} + \max\{\Pi^{D|SM}, \Pi^{D|S}\}, \tag{3.142}$$

which is always true for $0 < (\Delta E)^2 \leq \underline{\Phi}$ and $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Phi}$. Thus, $r^{RM} \geq r^{RE}$. Since $\Pi^{D|RM} \geq \Pi^{D|RE}$ in the considered intervals, the downstream party implements relational market if $r \leq r^{RM}$.
Now, consider the intervals $\underline{\Phi} < (\Delta E)^2 \leq \underline{\Theta}$ and $\overline{\Phi} < (\Delta E)^2$, where $\Pi^{D|RE} > \Pi^{D|RM}$. Suppose first that $r^{RE} \geq r^{RM}$, thereby implying $\tilde{\Pi}^{RE} = \max\{\Pi^{D|SM}, \Pi^{D|S}\}$ and $\tilde{\Pi}^{RM} = \Pi^{D|RE}$. Consequently, $r^{RE} \geq r^{RM}$ implies

$$2\left[\Pi^{D|RE} - \max\{\Pi^{D|SM}, \Pi^{D|S}\}\right] \geq \Pi^{D|RM} - \Pi^{D|RE}, \tag{3.143}$$

which is equivalent to

$$3\Pi^{D|RE} \geq \Pi^{D|RM} + 2\max\{\Pi^{D|SM}, \Pi^{D|S}\}. \tag{3.144}$$

This is satisfied for $\underline{\Phi} < (\Delta E)^2 \leq \underline{\Theta}$ and $\overline{\Phi} < (\Delta E)^2$ since $\Pi^{D|RE} \geq \Pi^{D|RM}$. Consequently, $r^{RE} \geq r^{RM}$. Due to the fact that $\Pi^{D|RE} \geq \Pi^{D|RM}$, the downstream party chooses relational employment if $r \leq r^{RE}$. Finally, the self-enforcement conditions can be obtained by substituting the downstream's respective fall-back profits emphasized by proposition 3.5 in r^{RM} and r^{RE}.
Next, consider the case $r^i < r \leq \widehat{r}^i$, $i = RM, RE$. Recall that $\Pi^{D|RM}(r) \geq \Pi^{D|RE}(r)$ in the intervals $0 < (\Delta E)^2 \leq \underline{\Psi}(r)$ and $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Psi}(r)$. Note that the cut offs $\underline{\Psi}(r)$ and $\overline{\Psi}(r)$ are valid for all $r \in (r^i, \widehat{r}^i]$, $i = RM, RE$.
Finally note that the values of relational contracts are decreasing in r for $r^i < r \leq \widehat{r}^i$, $i = RM, RE$. Consequently, it is necessary to identify the cut off interest rate $\bar{r}$ where the downstream party is indifferent between the respective relational contract

and the best spot alternative. Consider first relational employment. Hence, $\bar{r}$ implies $\Pi^{D|RE}(\bar{r}) = \tilde{\Pi}^{RE}$. Accordingly,

$$\frac{\bar{r}}{2}\left[\Delta I - \bar{r} + 2\left[\frac{1}{4}\left(\Delta I - \bar{r}\right)^2 + I_L - \tilde{\Pi}^{RE}\right]^{\frac{1}{2}}\right] + \tilde{\Pi}^{RE} = \tilde{\Pi}^{RE}, \tag{3.145}$$

which can be transformed to

$$\frac{1}{4}\left(\Delta I - \bar{r}\right)^2 + I_L - \tilde{\Pi}^{RE} = \frac{1}{4}\left(\Delta I - \bar{r}\right)^2. \tag{3.146}$$

Recall that for the collusion-free case $\tilde{\Pi}^{RE} = \max\{\Pi^{D|SM}, \Pi^{D|S}\} > I_L$. Consequently, (3.146) cannot be satisfied. This implies that $\widehat{r}^{RE}$ is the relevant cut off such that the downstream party receives $\max\{\Pi^{D|SM}, \Pi^{D|S}\}$ if $r > \widehat{r}^{RE}$. Finally, consider relational market, where $\bar{r}$ implies $\Pi^{D|RM}(\bar{r}) = \tilde{\Pi}^{RM}$. By applying the same approach as for relational employment, one can show that the downstream party receives $\max\{\Pi^{D|SM}, \Pi^{D|S}\}$ if $r > \widehat{r}^{RM}$.

Q.E.D.

Proof of Proposition 3.8.
Note that the downstream's optimal transaction may only change for $(\Delta E)^2 > \underline{\Theta}$, where supervision is the superior spot contract in a collusion-free environment, see proposition 3.5. Now consider the interval $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Phi}$, where $\Pi^{D|RM} \geq \Pi^{D|RE}$. Suppose first that $r^{RM} \geq r^D$, where r^D refers to the condition ensuring that the downstream party has no incentives to collude, see proposition 3.2. Recall that $r^S = \min\{r^W, r^D\}$. Hence, $r^{RM} \geq r^D$ would imply that $r^{RM} \geq r^S$. Next, observe that $r^{RM} \geq r^D$ is equivalent to

$$\frac{\Pi^{D|RM} - \tilde{\Pi}^{RM}}{\Delta I} \geq \frac{2\left[\Pi^{D|S} - \tilde{\Pi}^{S}\right]}{\Delta I}. \tag{3.147}$$

If $r^{RM} \geq r^D$, it follows $\tilde{\Pi}^{RM} = \max\{\Pi^{D|SM}, \Pi^{D|SE}\}$ and $\tilde{\Pi}^{S} = \Pi^{D|RM}$, where the particular value of $\tilde{\Pi}^{RM}$ depends on the downstream's best fall-back. As a consequence,

$$\Pi^{D|RM} - \max\{\Pi^{D|SM}, \Pi^{D|SE}\} \geq 2\Pi^{D|S} - 2\Pi^{D|RM}, \tag{3.148}$$

and equivalently,

$$3\Pi^{D|RM} \geq 2\Pi^{D|S} + \max\{\Pi^{D|SM}, \Pi^{D|SE}\}, \tag{3.149}$$

which is always true for $\underline{\Theta} < (\Delta E)^2 \leq \overline{\Phi}$ since $\Pi^{D|RM} \geq \Pi^{D|RE} > \Pi^{D|S}$. Therefore, $r^{RM} \geq r^D$, which further implies that $r^{RM} \geq r^S = \min\{r^W, r^D\}$. Since $\Pi^{D|RM} \geq \Pi^{D|S}$, the downstream party implements relational market if $r \leq r^{RM}$.
Next, consider the interval $\overline{\Phi} < (\Delta E)^2$ where $\Pi^{D|RE} > \Pi^{D|RM}$. Assume for a moment that $r^{RE} \geq r^D$. Again, $r^{RE} \geq r^D$ also implies $r^{RE} \geq r^S$ since $r^S = \min\{r^W, r^D\}$. Note that once the downstream party either reneged on β^* or colluded with the supervisor, the worker refuses to accept a contract from the downstream party based

on implicit agreements. Accordingly, supervision cannot be the fall-back for relational employment, and vice versa. This implies $\tilde{\Pi}^{RE} = \tilde{\Pi}^{S} = \max\{\Pi^{D|SM}, \Pi^{D|SE}\}$. As a consequence, $r^{RE} \geq r^{D}$ is equivalent to

$$\Pi^{D|RE} - \max\{\Pi^{D|SM}, \Pi^{D|SE}\} \geq \Pi^{D|S} - \max\{\Pi^{D|SM}, \Pi^{D|SE}\}, \tag{3.150}$$

which is always true for $\overline{\Phi} < (\Delta E)^2$ since $\Pi^{D|RE} > \Pi^{D|RM}$. Thus, $r^{RE} \geq r^{D}$, which additionally implies that $r^{RE} \geq r^{S} = \min\{r^{W}, r^{D}\}$. Since $\Pi^{D|RE} \geq \Pi^{D|S}$, the downstream party chooses relational employment if $r \leq r^{RE}$. Finally, the self-enforcement conditions can be achieved by substituting the downstream's respective fall-back profits from proposition 3.6 in r^{RM} and r^{RE}.

It is further necessary to identify whether the downstream party is better off by implementing collusion-proof supervision for $r > r^{RM}$ and $r > r^{RE}$, respectively. First consider the case $r^{W} < r^{D}$, which implies $\widehat{r}^{S} < r^{D}$. The preceding derivations in this proof indicate that the downstream party prefers either relational market or relational employment for every $r \leq r^{D}$. Next, consider the case $r^{W} \geq r^{D}$, which implies $\widehat{r}^{S} \leq r^{W}$. For this case, it is necessary to identify whether the downstream party chooses collusion-proof supervision for $r > r^{RM}$ and $r > r^{RE}$, respectively. First, consider the relevant intervals where relational market is the superior relational contract. Suppose for a moment that $r^{RE} \geq \widehat{r}^{S}$. This is equivalent to

$$\frac{\Pi^{D|RM} - \tilde{\Pi}^{RM}}{\Delta I} \geq \frac{\Pi^{D|S}(\widehat{r}^{S}) - \tilde{\Pi}^{D|S}}{\beta^{S*}(\widehat{r}^{S})}, \tag{3.151}$$

where $\beta^{S*}(\widehat{r}^{S})$ is the optimal bonus under supervision for $r = \widehat{r}^{S}$. If $r^{RM} \geq \widehat{r}^{S}$, it follows $\tilde{\Pi}^{RM} = \max\{\Pi^{D|SM}, \Pi^{D|SE}\}$ and $\tilde{\Pi}^{S} = \Pi^{D|RM}$, where the particular value of $\tilde{\Pi}^{RM}$ depends on the downstream's best fall-back. As a consequence,

$$\frac{\Pi^{D|RM} - \max\{\Pi^{D|SM}, \Pi^{D|SE}\}}{\Delta I} \geq \frac{\Pi^{D|S}(\widehat{r}^{S}) - \Pi^{D|RM}}{\beta^{S*}(\widehat{r}^{S})}, \tag{3.152}$$

which is equivalent to

$$\frac{\Pi^{D|RM}}{\Delta I} + \frac{\Pi^{D|RM}}{\beta^{S*}(\widehat{r}^{S})} \geq \frac{\Pi^{D|S}(\widehat{r}^{S})}{\beta^{S*}(\widehat{r}^{S})} + \frac{\max\{\Pi^{D|SM}, \Pi^{D|SE}\}}{\Delta I}. \tag{3.153}$$

Since $\Pi^{D|RM} > \Pi^{D|S}(r)$ and $\Pi^{D|RM} > \max\{\Pi^{D|SM}, \Pi^{D|SE}\}$ for the relevant intervals, it follows $r^{RM} \geq \widehat{r}^{S}$. Thus, the downstream party receives $\Pi^{D|RM}(r)$ for $r \leq \widehat{r}^{RM}$, and $\max\{\Pi^{D|SM}, \Pi^{D|SE}\}$, otherwise. Next, consider the intervals where relational employment is the superior relational contract. As demonstrated in proof of proposition 3.7, $r^{RE} \geq r^{RM}$ if $\Pi^{D|RE} > \Pi^{D|RM}$. Recall that $r^{RM} \geq \widehat{r}^{S}$. Consequently, $r^{RE} \geq \widehat{r}^{S}$ for the relevant intervals where $\Pi^{D|RE} > \Pi^{D|RM}$. As a result, the downstream party receives $\Pi^{D|RE}(r)$ for $r \leq \widehat{r}^{RE}$ in the relevant intervals, and $\max\{\Pi^{D|SM}, \Pi^{D|SE}\}$, otherwise. In sum, if supervision is collusion-proof, it is more profitable to implement either relational market or relation employment. Finally, the self-enforcement conditions can be obtained by substituting the downstream's respective fall-back profits from proposition 3.6 in $\widehat{r}^{RM}$ and $\widehat{r}^{RE}$.

Q.E.D.

Availability of Verifiable Signal.
First, it is essential that $\beta > 0$ in order to motivate the worker to implement $e_{Wi} > 0$ for at least one $i \in \{1, ..., n\}$. Thus, (3.69) is satisfied as long as (3.70) holds and therefore omits. Using these observations, the Lagrangian becomes

$$\mathcal{L}(\alpha, \beta) = I_L + \Delta I \beta \boldsymbol{\mu}^t \boldsymbol{\tau} - \alpha - \beta^2 \boldsymbol{\tau}^t \boldsymbol{\tau} + \lambda \left[\alpha + \frac{1}{2} \beta^2 \boldsymbol{\tau}^t \boldsymbol{\tau} \right] + \xi \alpha. \tag{3.154}$$

The first-order conditions are

$$-1 + \lambda + \xi = 0, \tag{3.155}$$

$$\Delta I \boldsymbol{\mu}^t \boldsymbol{\tau} + \beta \boldsymbol{\tau}^t \boldsymbol{\tau} (\lambda - 2) = 0, \tag{3.156}$$

and the complementary slackness conditions,

$$\lambda \left[\alpha + \frac{1}{2} \beta^2 \boldsymbol{\tau}^t \boldsymbol{\tau} \right] = 0, \tag{3.157}$$

$$\xi \alpha = 0. \tag{3.158}$$

First suppose $\lambda > 0$. In this case, $\alpha + \beta^2 \boldsymbol{\tau}^t \boldsymbol{\tau}/2 = 0$ due to (3.157). Since $\alpha \geq 0$, this would imply that $\alpha^* = 0$ and $\beta^* = 0$, and consequently, $\boldsymbol{e}^* = (0, ..., 0)^t$. Hence, $\lambda > 0$ cannot be a solution of this problem. For this reason $\lambda = 0$, i.e. the worker's participation constraint is not binding. Moreover, (3.155) implies that $\xi = 1$. Then, (3.158) leads to $\alpha^* = 0$. Re-arranging (3.156) with $\lambda = 0$ gives the optimal bonus

$$\beta^* = \frac{\Delta I \boldsymbol{\mu}^t \boldsymbol{\tau}}{2 \boldsymbol{\tau}^t \boldsymbol{\tau}}. \tag{3.159}$$

Since $\boldsymbol{\mu}^t \boldsymbol{\tau} = \|\boldsymbol{\mu}\| \|\boldsymbol{\tau}\| \cos\theta$, $\|\boldsymbol{\mu}\| = 1$, and $\boldsymbol{\tau}^t \boldsymbol{\tau} = \|\tau\|^2$, where θ is the angle between $\boldsymbol{\mu}$ and $\boldsymbol{\tau}$, β^* can be re-written as

$$\beta^* = \frac{\Delta I \cos\theta}{2 \|\boldsymbol{\tau}\|}. \tag{3.160}$$

Finally, substituting α^* and β^* in the downstream's objective function leads to

$$\Pi^{D|SE} = I_L + \frac{1}{4} (\Delta I)^2 \cos^2\theta. \tag{3.161}$$

Q.E.D.

Summary

The preceding three essays analyze several issues related to incongruent preferences for effort allocations in multi-task agency relations. The main emphasis is thereby on the provision of incentives aimed at improving the agent's effort allocation across relevant tasks.

The first essay illustrates how the provision of incentives incorporates agents' task-specific abilities. As demonstrated, task-specific abilities determine the efficiency of the agent's effort allocation in addition to the congruity of her performance evaluation. This essay highlights four important observations: First, when agents differ with respect to their task-specific abilities, it is optimal to provide them with various incentive contracts so as to take advantage of their respective abilities. This also includes the optimal aggregation of multiple performance measures. Second, the value of employing specific agents for particular tasks does not depend exclusively on their task-specific abilities, but is rather determined by the relationship between these abilities and the characteristics of the available performance measurement. Third, different types of agents can be equally valuable for performing the same set of tasks. However, their respective incentive contracts are nonetheless adjusted to their individual capabilities. This essay therefore provides a theoretical underpinning of the observation that agents are allocated to various jobs; and that they potentially receive different incentive contracts, even if they are employed for performing identical tasks and are assessed by the same information system. Finally, the value of information in multi-task agency relations depends—besides on their precision—on their information content with respect to the implemented effort allocation relative to the agent's task specific abilities. Accordingly, one cannot use a generic criterion to rank information in multi-task agencies, which is independent of the agents' characteristics. It is rather pivotal to refer to particular agents and their individual characteristics.

The second essay considers the costly acquisition of performance measures aimed at offering the agent more congruent incentives in the sense that she is motivated to implement a less distorted effort allocation. It contrasts two alternatives for the principal: (i) to centrally invest in the acquisition of additional performance measures; and (ii), to delegate the information acquisition to a supervisor, thereby inducing a second moral hazard problem. This essay demonstrates that the profitability of delegation depends on the relationship between the precision of the supervisor's performance evaluation, her relative measurement efficiency, and the congruence of the costless information system. This essay indicates that the two subsequent consequences arise if the supervisor's performance evaluation is sufficiently precise. First, the principal generally favors delegation if the costless information system is suffi-

ciently incongruent because it imposes less requirements on the supervisor's relative measurement efficiency. Accordingly, it is presumably that a potential supervisor is able to meet this prerequisite. Second, the principal prefers in general a centralized investment if the costless information system is sufficiently congruent. This is because delegation would impose high requirements on the supervisor's relative measurement efficiency in order to be favored by the principal. This in turn is less presumably to be achievable by a potential supervisor. The reversed implications apply if the supervisor's performance evaluation is sufficiently imprecise.

The third essay employs a multi-task agency framework to investigate the optimal transactions within and between firms, and the corresponding application of either explicit or implicit contracts. The main emphasis is on how (*i*) incongruent preferences for effort allocations between firms as an inefficiency of using the market; and (*ii*), potential side-contracting as an inefficiency of integrated productions, determine the optimal transactions and type of contracts. Incongruent preferences for effort allocations are thereby rooted in the discrepancy of requirements for the properties of produced and exchanged goods. This essay demonstrates that more congruent preferences impose additional costs for demanding firms to ensure relation-specific investments. In contrast, incongruent preferences do not affect the value of spot market transactions since the acquisition of less desirable products is compensated by lower transfer prices. This essay further illustrates that the effect of potential collusion is two-fold. First, collusion compromises the efficiency of integrated productions and therefore, may force firms to employ market transactions, even if integrated productions would have been more efficient otherwise. Second, potential collusion can be advantageous if it deteriorates the efficiency of integrated productions as best fall-back alternative for relational contracts. In this case, anticipated collusion can facilitate the achievement, or improve the efficiency, of superior relational contracts within and between firms. In addition, this essay illustrates that firms with access to sufficiently congruent measures about their employees' performance are more likely to adopt integrated productions rather than engaging in market transactions. However, receiving sufficiently congruent performance measures can improve the profitability of integrated productions as best fall-back alternative for relational contracts. Then, the availability of these measures can be disadvantageous in two ways: (*i*) it can compromise the feasibility of superior relational contracts; or (*ii*), it can deteriorate their profitability. Consequently, access to adequately congruent performance measures can impair the efficiency of relational contracts within and between firms.

Bibliography

R. N. Anthony and V. Govindarajan. *Management Control Systems*. Richard D. Irwin, Inc., eight edition, 1995.

B. J. Asch. Do incentives matter? the case of navy recruiters. *Industrial & Labor Relations Review*, 43(3):89–106, 1990.

G. Baker. The use of performance measures in incentive contracting. *American Economic Review*, 90(2):415–420, 2000.

G. Baker. Distortion and risk in optimal incentive contracts. *Journal of Human Resources*, 37(4):728–751, 2002.

G. Baker. Incentive contracts and performance measurement. *Journal of Political Economy*, 100(3):598–614, 1992.

G. Baker, R. Gibbons, and K. J. Murphy. Subjective performance measures in optimal incentive contracts. *Quarterly Journal of Economics*, 109(4):1125–1156, 1994a.

G. Baker, M. Gibbs, and B. Holmström. The internal economics of the firm: Evidence from personnel data. *Quarterly Journal of Economics*, 109(4):881–919, 1994b.

G. Baker, R. Gibbons, and K. J. Murphy. Bringing the market inside the firm? *American Economic Review, Papers and Proceedings*, 91(2):212–218, 2001.

G. Baker, R. Gibbons, and K. J. Murphy. Relational contracts and the theory of the firm. *Quarterly Journal of Economics*, 117(1):39–84, 2002.

R. D. Banker and S. M. Datar. Sensitivity, precision, and linear aggregation of signals for performance evaluation. *Journal of Accounting Research*, 27(1):21–39, 1989.

R. D. Banker and A. Thevaranjan. Goal congruence and evaluation of performance measures. Working Paper, University of Texas at Dallas, 2000.

P. Bolton and M. Dewatripont. *Contract Theory*. The MIT Press, Cambridge/Massachusetts, London/England, 2005.

J. A. Brickley and J. L. Zimmerman. Changing incentives in a multitask environment: Evidence from a top-tier business school. *Journal of Corporate Finance*, 7: 367–396, 2001.

C. Bull. The existence of self-enforcing implicit contracts. *Quarterly Journal of Economics*, 102(1):147–160, 1987.

R. M. Bushman, R. J. Indjejikian, and M. C. Penno. Private predecision information, performance measure congruity, and the value of delegation. *Contemporary Accounting Research*, 17(4):561–587, 2000.

P. O. Christensen and G. A. Feltham. *Economics of Accounting: Volume II - Performance Evaluation*. Springer, New York, 2005.

R. H. Coase. The nature of the firm. *Economica*, 4(16):386–405, 1937.

K. S. Corts. Teams vs. individual accountability: Solving multi-task problems through job design. Working Paper, Univerity of Toronto, 2005.

S. Datar, S. C. Kulp, and R. A. Lambert. Balancing performance measures. *Journal of Accounting Research*, 39(1):75–92, 2001.

R. M. Dawes and R. H. Thaler. Anomalies: Cooperation. *Journal of Economic Perspectives*, 2(3):187–197, 1988.

D. Demougin and C. Fluet. Monitoring versus incentives. *European Economic Review*, 45:1741–1764, 2001.

D. Demougin and C. Helm. Moral hazard and bargaining power. *German Economic Review (forthcoming)*, 2005.

A. S. Edlin and S. Reichelstein. Holdups, standard breach remedies, and optimal investment. *American Economic Review*, 86(3):478–501, 1996.

K. M. Eisenhardt. Agency theory: An assessment and review. *Academy of Management Review*, 14(1):57–74, 1989.

A. Faure-Grimaud, J.-J. Laffont, and D. Martimort. Transaction costs of collusion and organizational design. Research Paper Series, Center for Law, Economics and Organization, University of Southern California, 2001.

A. Faure-Grimaud, J.-J. Laffont, and D. Martimort. Collusion, delegation and supervision with soft information. *Review of Economic Studies*, 70(2):253–279, 2003.

G. A. Feltham and J. Xie. Performance measure congruity and diversity in multi-task principal/agent relations. *Accounting Review*, 69(3):429–453, 1994.

R. Gibbons. Incentives between firms (and within). *Management Science*, 51(1): 2–17, 2005.

R. Gibbons. Incentives in organizations. *Journal of Economic Perspectives*, 12(4): 115–132, 1998.

R. Gibbons and M. Waldman. Enriching a theory of wage and promotion dynamics inside firms. MIT Sloan School of Management Working Paper 4324-03, 2003.

R. Gibbons and M. Waldman. Task-specific human capital. *American Economic Review*, 94(2):203–207, 2004.

R. Gibbons and M. Waldman. A theory of wage and promotion dynamics inside firms. *Quarterly Journal of Economics*, 114(4):1321–1358, 1999.

M. Gibbs, K. A. Merchant, W. A. Van der Stede, and M. E. Vargus. Performance measure properties and incentives. IZA Discussion Paper No. 1356, 2004.

F. Gjesdal. Information and incentives: The agency information problem. *Review of Economic Studies*, 49(3):373–390, 1982.

A. Goodnough. Answers allegedly supplied in effort to raise test scores. *New York Times*, December 8, 1999.

S. J. Grossman and O. D. Hart. An analysis of the principal-agent problem. *Econometrica*, 51(1):7–46, 1983.

S. J. Grossman and O. D. Hart. The costs and benefits of ownership: A theory of vertical and lateral integration. *Journal of Political Economy*, 94(4):691–719, 1986.

M. Halonen. Reputation and the allocation of ownership. *The Economic Journal*, 112:539–558, 2002.

O. Hart and J. Moore. Property rights and the nature of the firm. *Journal of Political Economy*, 98(6):1119–1158, 1990.

R. M. Hayes and S. Schaefer. Implicit contracts and the explanatory power of top executive compensation for future performance. *RAND Journal of Economics*, 31 (2):273–293, 2000.

M. F. Hellwig and K. M. Schmidt. Discrete-time approximations of the holmström-milgrom brownian-motion model of intertemporal incentive provision. *Econometrica*, 70(6):2225–2264, 2002.

T. Hemmer, O. Kim, and R. E. Verrecchia. Introducing convexity into optimal compensation contracts. *Journal of Accounting and Economics*, 28(3):307–327, 2000.

D. Hofkins. Cheating 'rife' in national tests. *New York Times Educational Supplement*, June 16, 1995.

B. Holmström. Moral hazard and observability. *Bell Journal of Economics*, 10(1): 74–91, 1979.

B. Holmström. The firm as a subeconomy. *Journal of Law, Economics, & Organization*, 15(1):74–102, 1999.

B. Holmström and P. Milgrom. Aggregation and linearity in the provision of intertemporal incentives. *Econometrica*, 55(2):303–328, 1987.

B. Holmström and P. Milgrom. Multitask principal-agent analyses: Incentives contracts, asset ownership, and job design. *Journal of Law, Economics and Organization*, 7:24–52, 1991.

J. S. Hughes, L. Zhang, and J. Xie. Production externalities, congruity of aggregate signals, and optimal task assignments. *Contemporary Accounting Research*, 22(2): 393–408, 2005.

H. Itoh. Coalitions, incentives, and risk sharing. *Journal of Economic Theory*, 60: 410–427, 1993.

H. Itoh and H. Morita. Formal contracts, relational contracts, and the holdup problem. mimeo, 2005.

B. A. Jacob. Accountability, incentives and behavior: The impact of high-stakes testing in the chicago public schools. NBER Working Paper No. 8968, 2002.

B. A. Jacob and S. D. Levitt. Rotten apples: An investigation of the prevalence and predictors of teacher cheating. *Quarterly Journal of Economics*, 118(3):843–877, 2003.

P. L. Joskow. Asset specificity and the structure of vertical relationships: Empirical evidence. *Journal of Law, Economics, & Organization*, 4(1):95–117, 1988.

T. J. Kane and D. O. Staiger. The promise and pitfalls of using imprecise school accountability measures. *Journal of Economic Perspectives*, 16(4):91–114, 2002.

S. Kerr. On the folly of rewarding a, while hoping for b. *Academy of Management Journal*, 18(4):769–783, 1975.

A. S. Kessler. On monitoring and collusion in hierarchies. *Journal of Economic Theory*, 91:280–291, 2000.

S. K. Kim. Limited liability and bonus contracts. *Journal of Economics & Management Strategy*, 6(4):899–913, 1997.

S. K. Kim and Y. S. Suh. Ranking of accounting information systems for management control. *Journal of Accounting Research*, 29(2):386–396, 1991.

B. Klein and K. B. Leffler. The role of market forces in assuring contractual performance. *Journal of Political Economy*, 89(4):615–641, 1981.

B. Klein, R. G. Crawford, and A. A. Alchian. Vertical integration, appropriable rents, and the competitive contracting process. *Journal of Law and Economics*, 21(2):297–326, 1978.

S. P. Klein, L. S. Hamilton, D. F. McCaffrey, and B. M. Stecher. What do test scores in texas tell us? RAND Corporation, 2000.

F. Kofman and J. Lawarree. Collusion in hierarchical agency. *Econometrica*, 61(3): 629–656, 1993.

F. Kofman and J. Lawarree. On the optimality of allowing collusion. *Journal of Public Economics*, 61(3):383–407, 1996.

J.-J. Laffont and D. Martimort. Mechanism design with collusion and correlation. *Econometrica*, 68(2):309–342, 2000.

J.-J. Laffont and D. Martimort. Collusion under asymmetric information. *Econometrica*, 65(4):875–911, 1997.

J.-J. Laffont and M. Meleu. Reciprocal supervision, collusion and organizational design. *Scandinavian Journal of Economics*, 99(4):519–540, 1997.

R. A. Lambert. Contracting theory and accounting. *Journal of Accounting and Economics*, 32(1):3–87, 2001.

E. P. Lazear. Performance pay and productivity. *American Economic Review*, 90 (5):1346–1361, 2000a.

E. P. Lazear. The power of incentives. *American Economic Review*, 90(2):410–414, 2000b.

J. Levin. Relational incentive contracts. *American Economic Review*, 93(3):835–857, 2003.

A. Lindbeck and D. J. Snower. Multitask learning and the reorganization of work: From tayloristic to holistic organization. *Journal of Labor Economics*, 18(3):353–376, 2000.

R. L. Linn. Assessments and accountability. *Educational Researcher*, 29(2):4–16, 2000.

W. B. MacLeod. Complexity, contract and the employment relationship. Boston College Working Papers in Economics, No 342, 1996.

W. B. MacLeod and J. M. Malcomson. Implicit contracts, incentive compatibility, and involuntary unemployment. *Econometrica*, 57(2):447–480, 1989.

M. W. Maher, K. V. Ramanathan, and R. B. Peterson. Preference congruence, information accuracy, and employee performance: A field study. *Journal of Accouting Research*, 17(2):476–503, 1979.

D. Martimort. The life cycle of regulatory agencies: Dynamic capture and transaction costs. *Review of Economic Studies*, 66(4):929–947, 1999.

P. R. Milgrom. Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 12(2):380–391, 1981.

E. R. Moen and Å. Rosen. Performance pay and adverse selection. Working Paper, 2001.

P. Oyer. Fiscal year ends and nonlinear incentive contracts: The effect on business seasonality. *Quarterly Journal of Economics*, 113(1):149–185, 1998.

H. J. Paarsch and B. S. Shearer. The response of worker effort to piece rates: Evidence from the british columbia tree-planting industry. *Journal of Human Resources*, 34(4):643–667, 1999.

E. S. Park. Incentive contracting under limited liability. *Journal of Economics & Management Strategy*, 4(3):477–490, 1995.

D. G. Pearce and E. Stacchetti. The interaction of implicit and explicit contracts in repeated agency. *Games and Economic Behavior*, 23:75–96, 1998.

R. Pitchford. Moral hazard and limited liability: The real effects of contract bargaining. *Economics Letters*, 61:251–259, 1998.

C. Prendergast. The provision of incentives in firms. *Journal of Economic Literature*, 37(1):7–63, 1999.

C. Prendergast and R. H. Topel. Favoritism in organizations. *Journal of Political Economy*, 104(5):958–978, 1996.

R. Radner. Repeated principal-agent games with discounting. *Econometrica*, 53(5): 1173–1198, 1985.

R. Rees. The theory of principal and agent part i. *Bulletin of Economic Research*, 37(1):3–26, 1985.

B. Salanié. *The Economics of Contracts.* The MIT Press, Cambridge/Massachusetts, London/England, 1997.

H. J. Sapienza and A. K. Gupta. Impact of agency risks and task uncertainty on venture capitalist-ceo interaction. *Academy of Management Journal*, 37(6):1618–1632, 1994.

A. Schöttner. Relational contracts and job design. Working Paper Humboldt-University Berlin, 2005.

S. Shavell. Risk sharing and incentives in the principal and agent relationship. *Bell Journal of Economics*, 10(1):55–73, 1979.

S. Shavell. Contracts, holdup, and legal intervention. Harvard Law and Economics Discussion Paper No. 508, 2005.

K. Spremann. Agent and principal. In G. Bamberg and K. Spremann, editors, *Agency Theory, Information, and Incentives*, pages 3 – 37, Berlin et al., 1987.

B. M. Stecher and S. I. Barron. Quadrennial milepost accountability testing in Kentucky. CSE technical report no. 505, Center for the Study of Evaluation (CSE), 1999.

R. Strausz. Collusion and renegotiation in a principal-supervisor-agent relationship. *Scandinavian Journal of Economics*, 99(4):497–518, 1997.

L. G. Telser. A theory of self-enforcing agreements. *Journal of Business*, 53(1): 27–44, 1980.

J. Thomas and T. Worrall. Self-enforcing wage contracts. *Review of Economic Studies*, 55(4):541–553, 1988.

J. Tirole. Hierarchies and bureaucracies: On the role of collusion in organizations. *Journal of Law, Economics, & Organization*, 2(2):181–214, 1986.

J. Tirole. The multicontract organization. *Canadian Journal of Economics*, 21(3): 459–466, 1988.

J. Tirole. Collusion and the theory of organizations. In J.-J. Laffont, editor, *Advances in Economic Theory: Sixth World Congress*, volume 2. Cambridge University Press, 1992.

T. Tysome. Cheating purge: Inspectors out. *New York Times Higher Educational Supplement*, August 19, 1994.

B. Villadsen. Communication and delegation in collusive agencies. *Journal of Accounting and Economics*, 19:315–344, 1995.

O. E. Williamson. Transaction-cost economics: The governance of contractual relations. *Journal of Law and Economics*, 22(2):233–261, 1979.

Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

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